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6.—A CONVENIENT METHOD OF MEASURING RESISTANCES OF THE ORDER OF 10^{12} OHMS WITH A SENSITIVE BALLISTIC GALVANOMETER. By J. H. J. POOLE, Sc.D.

7.—REPORT OF THE IRISH RADIUM COMMITTEE FOR THE YEAR 1933. (Including Reports by OLIVER CHANCE, M.B., B.Ch., D.M.R.E., Richmond, Sir Patrick Dun's, Dr. Steevens', and Coombe Hospitals; W. G. HARVEY, M.D., F.R.C.P.I., Adelaide Hospital; W. J. MACHUGH, M.B., B.Ch., B.A.O., D.M.R.E., Galway Central Hospital; OSWALD J. MURPHY, M.B., B.Ch., St. Vincent's Hospital, Dublin.)

8.—THE CHEMICAL CONSTITUENTS OF LICHENS FOUND IN IRELAND. *Buellia canescens.—Part I.* By T. J. NOLAN, B.A., D.Sc., University College, Dublin. (Plate 2.)

9.—DERANGEMENT OF THE DIGESTIVE PROCESSES IN THE MILK-FED CALF DUE TO ABNORMAL CURD FORMATION IN THE FOURTH STOMACH. By E. J. SHEEHY, B.Sc., F.R.C.Sc.I., University College, Dublin.

10. THE APPROXIMATE DETERMINATION OF THE VIBRATION OF BEAMS AND THE WHIRLING OF SHAFTS. By H. H. JEFFCOTT, Sc.D.

11.—INVESTIGATIONS OF THE CRYOSCOPY OF MILK. By J. J. RYAN and G. T. PYNE (Department of Dairy Chemistry, University College, Cork). [Communicated by PROFESSOR J. REILLY.]

12.—THE INFLUENCE OF THE STAGE OF LACTATION ON FAT ESTIMATIONS BY THE GERBER METHOD. By J. LYONS and M. O'SHEA, Dairy Technology Department, University College, Cork. [Communicated by MR. E. J. SHEEHY.]

[*Authors alone are responsible for all opinions expressed in their Communications.*]

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No. 6.

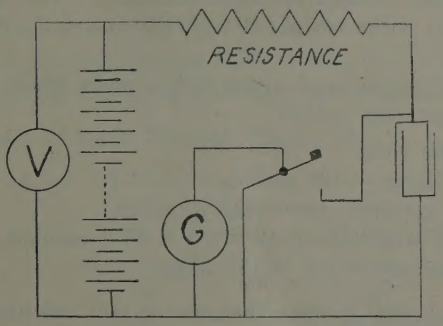
A CONVENIENT METHOD OF MEASURING RESISTANCES OF THE ORDER OF 10^{12} OHMS WITH A SENSITIVE BALLISTIC GALVANOMETER.

By J. H. J. POOLE, Sc.D.

[Read May 24. Published separately June 22, 1934.]

The method of measuring high resistances with a sensitive current galvanometer is limited in its range by the voltage drop which can be applied to the resistance, and the sensitivity of the galvanometer. Unless a very large voltage and an ultra-sensitive galvanometer are employed, it is unsuitable for resistances much greater than 10^{11} ohms. Such a galvanometer may not be available, and there is an objection to using too high a voltage to measure the resistance, which may be different under smaller voltages. Another method which has been used is to measure the rate of loss of charge or potential of a charged condenser when shunted by the high resistance to be measured. The disadvantage of this method is that the insulation resistance of the condenser must itself be, at least, as great as the resistance to be measured, and such condensers are not available in most physical laboratories. The method described in this paper is a very obvious way of getting round both these difficulties, and is very simple to fit up with ordinary standard apparatus. Since only the high resistance itself has a large voltage drop across it, no very special precautions as regards insulation will be found necessary.

The figure shows the circuit used. A high tension battery, whose voltage can be measured by a suitable voltmeter, charges a standard capacity through



the high resistance. When the capacity has been charging for a known time, it is discharged by means of an ebonite Morse key through the ballistic galvanometer. The key is arranged to keep the galvanometer short-circuited except

when it is depressed. The *modus operandi* is to depress the key to discharge the condenser, then release it, at the same instant starting a stop-watch; allow the condenser to charge for a suitable time, and then depress the key again, and observe throw of galvanometer. The condenser used was a standard $\frac{1}{3}$ microfarad mica condenser, the galvanometer a Tinsley moving-coil, whose sensitivity under the experimental conditions was $5{\cdot}5 \times 10^{-10}$ coulomb per scale division. Thus a P.D. of half a volt across the condenser gave a throw of more than 300 divisions. Usually the maximum P.D. across the condenser was about 0·2 volt, and under these circumstances it was found that the leakage of the condenser and Morse key were inappreciable. To avoid any trouble with direct leakage from the H.T. battery, it was found sufficient to place it on the floor, and bring one lead to the high resistance on the bench. This lead must not touch anything *en route*. The galvanometer stood on an earthed metal stand.

The increase of sensitivity gained by this method can easily be seen. Thus if k is the current sensitivity, the quantity sensitivity is approximately $\frac{kT}{2\pi}$ where T is the periodic time, about 8 sec. for this galvanometer. The actual figures were $3{\cdot}3 \times 10^{-10}$ amp. per div., and $5{\cdot}2 \times 10^{-10}$ coulomb per div. Thus, if the condenser is allowed to charge up for $\frac{5{\cdot}2}{3{\cdot}3} \times 100$, i.e., 157 sec., the ballistic throw will be 100 times as great as the steady deflexion. As the leakage of the condenser can easily be made very small, much longer times can be employed if desired, the main limit being the patience of the observer. To get an amplification of 1,000 times it would, however, be necessary to wait nearly half-an-hour, and probably extra precautions as to insulation would be required.

In the above paragraph I have neglected the decrease in P.D. across the high resistance due to the charging of the condenser, and assumed that there is a constant current through the high resistance. This is justifiable, as the H.T. voltage is very large compared with the maximum P.D. across the condenser.

Some actual figures obtained with a carbon smoke film on Pyrex glass tubing are given to illustrate the method.

H.T. = 400 volts.
Time of Charge = 120 seconds.
Throw of galvanometer = 118 divisions.
Sensitivity of galvanometer = $5{\cdot}5 \times 10^{-10}$ coulomb per division.
Hence resistance = $7{\cdot}4 \times 10^{11}$ ohms.

In conclusion, I wish to state that, as far as I can ascertain, this method has not been described before; but it seems such an obvious one that I should be very chary of claiming it as original.

No. 7.

REPORT OF THE IRISH RADIUM COMMITTEE FOR THE YEAR 1933.

[Read MAY 29. Published separately JULY 6, 1934.]

THE Committee has sustained a severe loss during the year through the death of Professor John Joly, SC.D., F.R.S., whose invention of the radon capillary method of radium therapy had rendered the work of the Irish Radium Institute possible. The establishment of the Institute in 1914 was chiefly due to his efforts, and he had remained to the end a very active and highly valued member of the Committee.

306 batches of radon tubes, containing 12,996 millicuries of radon, were issued during the year for therapeutic purposes. This is a small reduction on the amount issued during 1932.

Mr. G. Enright and Mr. H. Pringle were appointed to the Radium Exhibitions falling vacant during the year.

Reports from some of the largest users of radon supplied by the Institute are printed below.

Returns of the numbers of cases treated with radon alone, or with radon in conjunction with Surgery and (or) X-Rays, have also been received from the following hospitals:—

City of Dublin Skin and Cancer Hospital, Dublin ...	20 cases.
Coombe Hospital, Dublin	2 „
Rotunda Hospital, Dublin	5 „
St. Anne's Hospital, Dublin	59 „
St. Ultan's Infant Hospital, Dublin	1 case.

The total number of cases recorded as treated during the year is 466.

REPORT BY OLIVER CHANCE, M.B., B.CH., D.M.R.E.,

Richmond, Sir Patrick Dun's, Dr. Steevens', and Coombe Hospitals.

List of Cases Treated (Total, 191).

Malignant, 135.

Rodent Ulcer	43	Carcinoma Gum	2
Carcinoma Skin	21	„ Cervix Uteri ...	7
„ Lip	13	„ Breast	2
„ Tongue	19	„ Penis	1
„ Cheek	2	„ Oesophagus ...	2
„ Palate	5	Branchial Carcinoma	2
„ Floor of Mouth ...	5	Sarcoma	2
„ Fauces, Tonsils, and Pharynx ...	8	Hodgkin's Disease	1

Non-Malignant, 56.

Tubercular Peritonitis ...	14	Painful Scars	2
Toxic Goitre	15	Naevi	13
Uterine Hæmorrhage	6	Warts	3
Lupus	3		

Notes.

No changes in technique were made during the past year.

Carcinoma Mammae. — These cases are still treated by a combination of surgery and X-ray therapy. Radium was only used in the treatment of two cases in which recurrence was localised.

Toxic Goitre.—In most of these cases the patient is put to bed and a radium collar used. Three months later a course of X-rays is given. This is repeated at three monthly intervals until the patient is free from disease. As a rule, 3–4 courses of X-ray treatment are necessary.

Two rather interesting tongue cases were seen during the year. One patient had a very indurated mass the size of a walnut in the mid-line far back on the dorsum of his tongue. Radium was implanted into it, and at the same time biopsy was performed. The pathologist reported the tumour to be a fibroma. The radium treatment had no effect whatever. The tumour has grown steadily and is now causing symptoms of obstruction. Owing to its situation excision is impossible.

The second patient was first seen in February. He had been diagnosed at two other clinics as a hopeless case of lingual cancer with inoperable glands. Two-thirds of the tongue was involved by a foul indurated ulcer. Tongue mobility was nil. There were fixed masses in the right sub-maxillary and sub-mental triangles, and enlarged glands along the left sterno-mastoid muscle. The patient's teeth were removed because they were irritating the growth, and he was told that his condition was too advanced to benefit from further treatment.

The patient returned to hospital in September. His tongue was completely healed with great loss of substance, and the glands along the sterno-mastoid and those in the sub-maxillary region had disappeared, but there was a fungating mass in the sub-mental triangle. The pathologist reported this to be a squamous carcinoma of high maturity. Radium was implanted into it, and when last seen 4 months later the mass, while still active, was considerably reduced in size. The mouth was still free from disease.

It would appear that in this case spontaneous local cure of a cancer of the tongue had occurred.

Malignant Cases.

Rodent Ulcers, 43.

Total number of cases seen, 44. Treated, 43. Well, 35. Improved, 4. Untraced, 4. Unsuitable, 1.

Carcinoma Skin, 21.

Total number of cases seen, 24. Treated, 21. Well, 14. Improved, 2. Unimproved, 1. Untraced, 4. Unsuitable, 3.

Buccal Carcinoma, 54.

Total number of cases seen, 77. Treated, 54. Unsuitable, 23.

	No. of cases.	Well.	Primary well, but glands involved.	Unimpr.	Untraced.	Still under Treatment.	Dead.
Carcinoma Lip, 13.							
*Group 1	8	4	1	1	2	—	—
„ 2	4	2	1	—	—	—	1
„ 3	1	—	1	—	—	—	—
Carcinoma Tongue, 19.							
Group 1	13	8	—	2	—	2	1
„ 2	5	2	2	—	1	—	—
„ 3	1	—	—	—	—	—	1
Carcinoma Cheek, 2.							
Group 1	1	1	—	—	—	—	—
„ 2	1	1	—	—	—	—	—
„ 3	—	—	—	—	—	—	—
Carcinoma Palate, 5.							
Group 1	4	2	—	1	—	—	1
„ 2	1	—	—	—	—	—	1
„ 3	—	—	—	—	—	—	—
Carcinoma Floor of Mouth, 5.							
Group 1	4	1	1	1	—	—	1
„ 2	1	1	—	—	—	—	—
„ 3	—	—	—	—	—	—	—
Carcinoma Fauces, Tonsil, & Pharynx, 8.							
Group 1	3	1	—	1	—	—	1
„ 2	3	1	—	—	1	—	1
„ 3	2	—	—	—	1	—	1
Carcinoma Gum, 2.							
Group 1	2	1	1	—	—	—	—
„ 2	—	—	—	—	—	—	—
„ 3	—	—	—	—	—	—	—

*No gland involvement Group 1.
Glands enlarged, but mobile Group 2.
Glands enlarged and inoperable Group 3.

Carcinoma Cervix Uteri, 7.

Number of cases seen, 15. Treated, 7. Unsuitable, 8.

	No. of cases.	Well.	Improved, but disease still active.	Unimproved.	Untraced.	Dead.
Operable ...	1	1	—	—	—	—
Borderline ...	3	2	—	—	1	—
Inoperable ...	3	—	2	—	1	—
Hopeless ...	8	—	—	—	—	—

Carcinoma Breast, 18.

Number of cases seen, 23. Treated, 18. Unsuitable, 5.

	No. of cases.	Free from Disease.	Symptoms relieved.	Disease active.	Unimproved.	Dead.
Prophylactic irradiation ...	11	8	—	2	—	1
Palliative irradiation ...	2	—	2	—	—	—
Recurrent Cases ...	5	2	—	—	2	1

Carcinoma Penis, 1.

Cases seen, 2. Treated, 1. Well, 1.

Carcinoma Oesophagus, 2.

Seen, 2. Treated, 2. Improved, 2.

Sarcomas, 4.

Cases seen, 10. Treated, 4. Free from disease, 1. Dead, 1. Unimproved, 1.

Hodgkin's Disease, 1.

Improved, 1.

Cerebral Tumours, 1.

Treated with X-rays, 1. Dead 1.

Branchial Carcinoma, 2.

Seen, 2. Treated, 2. Improved, 2.

Non-Malignant Cases, 56.

Tubercular Peritonitis, 14.

Well, 6. Improved, 2. Still under treatment, 3. Dead, 3.

Toxic Goitre, 15.

Well, 3. Still under treatment, improved, 9. Untraced, 2. Unimproved, 1.

Fibrosis Uteri and *Menopausal Hæmorrhage,* 6.

Well, 5. Untraced, 1.

Amenorrhœa, 1.

Unimproved, 1. A stimulating dose of X-rays, 15 per cent. to each ovary, was given without result.

Lupus, 3.

Well, 1. Improved, 1. Untraced, 1.

The disease in all cases was comparatively localised; treatment was by means of screened surface irradiation.

Painful Scars, 2.

Well, 1. Improved, 1.

Naevi, 13.

Well, 4. Still under treatment, greatly improved, 9.

Warts, 3.

Well, 3. In all cases the warts were situated close to finger nails, and surgery was decided to be impracticable.

REPORT BY W. G. HARVEY, M.D., F.R.C.P.I.

Cases treated with Radon at Adelaide Hospital during 1933.

This treatment was only commenced on 18th December, 1933, and therefore the report is extremely meagre.

Only 3 cases were treated:—

Case 1.—H.R. Old epithelioma previously treated with carbon dioxide snow and X-rays. Never completely healed. Treated by surface application of needles. Total mc.s destroyed, 2·04. A month afterwards appeared completely cured.

Case 2.—P.R.S. Epithelioma, 2 needles inserted; mc.s destroyed, 1·36. A month afterwards very greatly improved. ? cured.

Case 3.—A.H. Fungating epithelioma, 8 needles inserted; mc.s destroyed, 13·6. Tumour has disappeared, but it is yet too soon to judge finally of result.

REPORT BY W. J. MACHUGH, M.B., B.CH., B.A.O., D.M.R.E.,

Galway Central Hospital.

During 1933, 6 cases received treatment with radon.

The cases were as follows:—

1 case inoperable Carcinoma of Breast.
1 „ recurrent mixed Parotid Tumour.

1 case Rodent Ulcer.
1 „ Carcinoma of Tongue, with glands in neck.
1 „ Carcinoma of Cervix (inoperable).
1 „ malignant Thyroid.

5 received radon only, and 1 received surgery and radon.

The radon figures apply only to cases seen since August 1st, when I took over charge of the X-ray department here.

Report by Oswald J. Murphy, M.B., B.Ch.,
St. Vincent's Hospital, Dublin.
1933.

List of Cases Treated (Total, 179).

Malignant, 95.

	Number seen.	Number Treated.	Died.	Untraced.	Under Treatment.	Well.
Carcinoma.						
Lip	6	6	–	1	4	1
Tongue	5	3	1	–	1	1
Palate	2	2	–	–	2	–
Tonsil	3	3	1	1	1	–
Pharynx	1	–	–	–	–	–
Larynx	3	1	1	–	–	–
Oesophagus ...	2	1	–	1	–	–
Branchial Cleft ...	3	2	1	–	1	–
Breast	4	2	–	–	1	1
„ Secondaries ...	8	8	1	2	4	1
Penis	1	1	–	–	–	1
Rectum	4	–	–	–	–	–
Anus	1	1	–	–	–	1
Uterine Cervix ...	4	3	–	1	1	1
„ Body ...	1	–	–	–	–	–
Skin Surfaces ...	26	17	–	6	6	5
Glands in Neck ...	9	7	–	1	5	1
„ „ Axilla ...	3	3	–	1	2	–
Rodent Ulcers ...	35	35	–	7	12	16
	—	—	–	—	—	—
	121	95	5	21	40	29

Non-Malignant, 84.

	Treated.	Under Treatment.	Died.	Improved.	Well.
Exophthalmic Goitre	2	–	–	2	–
Enlarged Spleen	2	–	–	2	–
Hodgkin	10	3	3	3	1
T.B. Cervical Glands	10	2	–	6	2
Uterine Hæmorrhages ...	2	–	–	1	1
Angiomata	17	6	–	3	8
Lupus Vulgaris	3	1	–	2	–
Lupus Erythematosus	2	1	–	–	1
Keloids	9	2	–	4	3
Warts	21	5	–	1	15
Corns	6	2	–	2	2
	—	—	–	—	—
	84	22	3	26	33
	—	—	–	—	—

Cases treated by Radium or by Surgery and Radium.

	Radium.	Radium and Surgery.
Carcinoma.		
Lip	5	1
Tongue	3	
Palate	2	
Tonsil	2	1
Pharynx, Larynx, and Oesophagus ...	2	
Branchial Cleft ...		2
Breast	1	1
„ Secondaries	8	
Carcinoma.		
Penis	1	
Anus	1	
Uterine Cervix ...	3	
Skin Surfaces ...	17	
Glands in Neck ...	5	2
Glands in Axilla ...	2	1
Rodent Ulcers ...	35	

During the year under review the various techniques employed in previous years have, with slight modification, been continued. In one case of Carcinoma of the floor of the mouth and under surface of the tongue, in addition to implantation of needles, subsequent removal of glands in neck, and placing of needles throughout the area, a course of Fischera 365 serum was given twice weekly for 19 injections. The injections were started in the last week of December, and in the ensuing weeks several interesting observations were made. The patient always felt depressed and "slack" the day following each injection.

The operation scar, which had healed perfectly since the second week in September, after the second injection became very tender, inflamed, and tense

in one spot—this subsequently broke down. Long tenacious grey sloughs came away, followed by a copious watery discharge. The sloughs varied in length up to 5 inches and came away like ribbon.

This continued until the patient died.

Summary of Results for 4 years.

Malignant Cases:—Seen, 477. Treated, 417.

Carcinoma.	Number Seen.	Number Treated.	Died in 1st year.	Died in 2nd year.	Died in 3rd year.	Died in 4th year.	Untraced.	Under treatment.	Well.
Lip ...	22	21	2	4	2	1	4	4	4
Tongue ...	32	20	10	4	1	1	2	1	1
Palate ...	10	10	3	1	1	–	1	2	2
Tonsil ...	10	10	5	1	1	–	1	1	1
Pharynx ...	7	5	4	1	–	–	–	–	–
Larynx ...	11	8	6	2	–	–	–	–	–
Thyroid ...	4	4	2	1	–	–	1	–	–
Oesophagus ...	5	3	3	–	–	–	–	–	–
Branchial Cleft	12	10	8	1	–	–	–	1	–
Breast ...	18	11	2	4	1	1	1	1	1
„ Secondaries	20	20	7	4	2	–	1	4	2
Penis ...	3	3	1	–	–	–	–	1	1
Rectum ...	8	3	2	1	–	–	–	–	–
Anal Orifice ...	3	3	–	1	–	–	–	–	2
Uterine Cervix	19	18	4	4	1	2	4	1	2
„ Body	4	2	–	–	–	–	2	–	–
Skin Surfaces	109	89	8	11	2	2	21	6	39
Glands in Neck	31	26	6	8	2	–	4	5	1
Glands in Axilla	11	10	3	2	1	–	2	2	–
Melanotic Sarcoma	4	4	4	–	–	–	–	–	–
Sarcoma Maxillary Antrum ...	2	2	1	–	–	–	1	–	–
Rodent Ulcers ...	132	135	–	–	–	–	7	12	116
	477	417	81	50	14	7	52	41	172

(Signed) Edward Coey Bigger,
Chairman, Irish Radium Committee.

No. 8.

THE CHEMICAL CONSTITUENTS OF LICHENS FOUND IN IRELAND.

Buellia canescens.—Part I.

By T. J. NOLAN, B.A., D.Sc.,
University College, Dublin.

(Plate 2.)

[Read May 29. Published separately July 20, 1934.]

The lichen *Buellia canescens* is found in many districts in Ireland, but rarely in the fertile condition. The specimen investigated in the present paper was found on the wall of Mount Druid, Killiney, Co. Dublin, and was collected in January, 1934. It was identified by Dr. P. O'Connor of the National Museum.

This lichen under the name *Diploicia* Dicks (*Catolechia canescens* Th. Fries) has been investigated by Zopf (Annalen der Chemie, **336**, p. 58), who collected his material from the walls of the Castle of Bentheim in Westphalia. Zopf found in the lichen three substances. Diploicin, the main constituent, crystallised in brown coral clusters m.p. 225° C.; the brown colour was believed to be a characteristic of the material. Catolechin, a minor constituent, crystallised in long silken needles, sometimes arranged in cushion form; it melted at 215° C. with previous softening at 210° C. It was very similar in properties to Diploicin, and, apart from its crystalline form and melting point, differed from the latter only in being somewhat more easily soluble in organic solvents. The third constituent was identified by Zopf as Atranorin. It had the characteristic crystalline form of this latter substance, and on heating with alcohol gave a product m.p. 113° C. believed to be hæmatommic ester. No analytical data are given by Zopf in his paper. In our experiments we isolated as the major constituent, Diploicin. This material we obtained eventually in the form of colourless crystals of coral-like structure (Pl. 2, Fig. 1), so that, although a brown colour tends to persist, this can be removed and is consequently not a characteristic of the substance. The product had all the characteristics described by Zopf. It is soluble with difficulty in cold benzene and alcohol, insoluble in concentrated sulphuric acid, insoluble in potash, and neutral in reaction, and gave no coloration with ferric chloride. We have obtained from

it by a mild treatment with alkali or a base a colourless product soluble in alkali and giving no reaction with ferric chloride. Diploicin contains no nitrogen or sulphur but contains over 30 per cent. chlorine.[1] It contains no methoxyl group, nor does it react with aniline in alcoholic solution, so that an aldehyde group does not appear to be present. With acetic anhydride and sodium acetate it gives a mono-acetate derivative (Fig. 3). We are continuing our investigations of this interesting substance, and at present tentatively put forward as working formulae $C_{15}H_{10}O_4Cl_4$ or $C_{16}H_{10}O_5Cl_4$.

We have not succeeded in isolating Catolechin from the lichen. We have noted that impure Diploicin tends to crystallise in needle-like tufts, and that dilute solutions of Diploicin will sometimes deposit long silken needlar crystals (Fig. 2). We have analysed such crystals, and have found them to give results identical with those of Diploicin. In our opinion Catolechin and Diploicin are identical.

We have isolated a further constituent, Fig. 4, similar in properties to the material identified by Zopf as Atranorin. Our substance is, however, not Atranorin. It has most of the properties of the latter, but contains no methoxyl group. The analytical data are different from those of Atranorin, and the empirical formula suggested $C_{18}H_{16}O_9$ could explain the production of Hæmatommic acid by hydrolysis; the nature of this substance is being further investigated. In addition we have isolated *Mannitol* and a well-defined crystalline water-soluble organic material which does not melt.

Experimental.

300 g. of the air-dried lichen were exhaustively extracted with ether and the ethereal solution decanted and filtered. The clear ethereal extract was evaporated to dryness. During the course of the evaporation white needle-shaped crystals separated out, and finally a brownish solid remained behind. The lichen was then extracted with acetone, but this yielded only a small amount of an amorphous reddish product, from which nothing definitely crystalline was isolated. A final extraction of the lichen with ethyl alcohol (500 c.c.) gave a tarry product which on treatment with a small amount of alcohol left a brownish insoluble product, which was warmed with methyl alcohol. The methyl alcohol solution on cooling deposited colourless crystals in the form of well-defined rectangular plates. This material was soluble in water, gave no reaction with ferric chloride, had no well-defined taste, gave no precipitate on treating an aqueous solution with acid, did not melt below 250° C., and on ignition left no apparent ash. It was not further investigated. The yield was small.

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[1] Koller and Pöpl ("Nature," May 26th, p. 808) announce the discovery of a lichen substance, which they state contains about 4 per cent. (*sic*) chlorine, and is believed by them to be chlorinated Atranorin. *Vide* also author's letter in "Chemistry and Industry," June 8th, 1934, p. 512.

The ethyl alcoholic solution from the tarry product on treatment with an equal volume of acetone gave on standing a brownish precipitate which on crystallising twice from methyl alcohol gave colourless needles: m.p. 166–167° C. This material had all the properties of mannitol, and when mixed with the latter showed no depression in the melting point.

The residue obtained on evaporating the ethereal extract weighed 9 g. It was extracted with petroleum ether, which gave a yellowish solution with a green fluorescence. This solution contained only a small amount of resinous material. The portion insoluble in petroleum was ground up with about 70 c.c. of ether. This removed a considerable amount of dark brown product, and the residual material was much lighter in colour. It weighed 5·5 g. This product was crystallised from 60 c.c. benzene and yielded 4·3 g. of material. This material on warming with 110 c.c. ethyl alcohol left an insoluble product *A* (wt. = 0·3 g.), which after repeated crystallisation from benzene, in which it was soluble with difficulty, gave a crystalline product (Fig. 4), of a pale yellow colour, very similar to Atranorin in crystalline appearance: m.p. 205–206° C. The alcoholic solution referred to above on cooling deposited a mass of needle-like crystals (wt. 2·7 g.). This material was again recrystallised from alcohol, but was still contaminated with some of the product *A*. On further repeated crystallisation from benzol, the material was eventually obtained free from product *A*, and crystallised as coral-like aggregates (Fig. 1), as described by Zopf, but free from colour: m.p. 232° C. All mother liquors were exhaustively worked up in order to endeavour to isolate a product corresponding to Zopf's Catolechin, but in no case was any product other than Diploicin and the product *A* obtained.

Analysis of Diploicin.

All analyses were carried out on material dried at 110° C. in high vacuum over P_2O_5, and represent various preparations. Results are by Dr. A. Schoeller.

			Percentage		
Wt. of Substance	CO_2	H_2O	C	H	Cl
5·032 mg.	8·395 mg.	1·110 mg.	45·50	2·45	—
5·130 „	8·570 „	1·150 „	45·58	2·49	—
2·964 „ gave	0·995 „ Cl.		—	—	33·56

Methoxyl determination—completely negative.

Molecular wt. determination.

0·242 mg. in 2·345 mg. camphor	11·0° Δ	M.W. = 376
0·272 „ „ 3·140 „ „	9·2° Δ	M.W. = 377

Treatment of Diploicin with Hydriodic acid.

Diploicin was treated with hydriodic acid in accordance with the Zeisel method, and on crystallising the residual material from benzene a product was obtained, m.p. 232° C., which showed no depression of melting point on mixing with the original material.

Wt. of Substance	CO_2	H_2O	Percentage C	Percentage H
5·135 mg.	8·550 mg.	1·080 mg.	45·41	2·34
4·590 „	7·650 „	1·040 „	45·45	2·52

In the crystallisation of the above product silken needle-like crystals were obtained when the mother liquors were allowed to stand (Fig. 2), m.p. 232° C. This material gave the following results on analysis.

Wt. of Substance	CO_2	H_2O	Percentage C	Percentage H	Percentage Cl
4·716 mg.	7·820 mg.	1·020 mg.	45·22	2·40	—
3·404 „ gave 1·131 „ Cl.			—	—	33·23

	C.	H.	Cl.	M.W.
$C_{15}H_{10}O_4Cl_4$ requires	45·45	2·52	35·86	396
$C_{16}H_{10}O_5Cl_4$ „	45·29	2·36	33·49	424

Properties of Diploicin.

Soluble with difficulty in ether, alcohol, and benzene; readily soluble in chloroform or acetone. Insoluble in sulphuric acid and dilute caustic potash. An alcoholic solution gives no colour reaction with ferric chloride. On treatment with dilute alcoholic potash it gives a product soluble in potash. This product gives no reaction with ferric chloride. Diploicin crystallises from alcohol in needle-like crystals, which contain combined alcohol not removed by heating in vacuo at 110° C. but removable by recrystallising from benzene.

Treatment of an alcoholic solution of Diploicin with aniline does not result in interaction, the Diploicin being recovered unchanged.

Action of Acetic Anhydride and Sodium Acetate on Diploicin.

0·2 g. of Diploicin was heated with 6 c.c. of acetic anhydride and 0·6 g. fused sodium acetate for 15 minutes, then cooled, poured into 25 c.c. water, and allowed to stand overnight. The product was filtered, washed with water, and dried in a dessicator. It was then dissolved in 3 c.c. benzene, filtered, and the solution allowed to stand for 24 hours. The material crystallised in flat thin plates (Fig. 3), m.p. 234–235° C. A mixture with Diploicin melted at 206° C.

Wt. of Substance	CO_2	H_2O	Percentage C	Percentage H	Percentage Cl
5·173 mg.	8·815 mg.	1·330 mg.	46·47	2·85	—
2·978 „	5·090 „	0·760 „	46·64	2·86	—
3·159 „ gave 1·022 „ Cl.			—	—	32·35

0·212 mg. in 2·210 mg. camphor – 10·5° Δ M.W. 365
0·366 „ „ 4·080 „ „ – 9·7° Δ M.W. 370

	C	H	Cl	M.W.
$C_{15}H_9O_4Cl_4(CH_3CO)$ requires	46·58	2·74	32·42	438
$C_{16}H_9O_5Cl_4(CH_3CO)$ „	46·35	2·58	30·5	466

*Analysis of Product A.**

The analyses are of two different preparations.

Wt. of Substance	CO_2	H_2O	Percentage C	Percentage H
4·340 mg.	9·110 mg.	1·730 mg.	57·25	4·43
5·162 „	10·825 „	2·060 „	57·20	4·43

Methoxyl determination negative.

Molecular wt. determination.[2]

0·400 mg. in 3·840 mg. camphor 19·2° Δ 216 M.W.
0·400 „ „ 6·030 „ „ 12·0° Δ 221 M.W.
0·346 „ „ 3·690 „ „ 17·2° Δ 218 M.W.
0·354 „ „ 3·270 „ „ 20·0° Δ 216 M.W.

$C_{18}H_{16}O_9$ requires C. 57·44 H. 4·25 M.W. 376

Properties of Product A.

A pale yellow solid crystallising as shown (Fig. 4), m.p. 205–206° C. Soluble with difficulty in ether, benzene, and alcohol. Insoluble in sodium bicarbonate; soluble in caustic potash giving a yellow solution. An alcoholic solution gives with ferric chloride a port-wine colouration.

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[2] We have come to the conclusion that simple depsides of the Atranorin type give values for the molecular weight by the Rast method which are much too low. For example, Sphaerophorin (M.W. 416) gave a value of 204. The above molecular weights correspond to a formula $C_{10}H_{10}O_5$, which requires C. 57·1, H. 4·76.

**Note added in Press.*—Product *A* has been found to contain 6·8 per cent. chlorine.

FIG. 1.

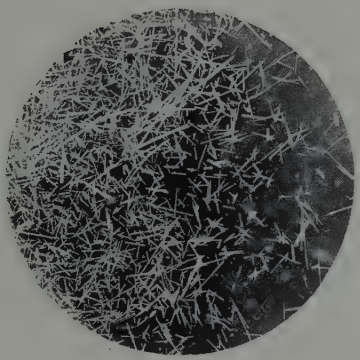


FIG. 2.

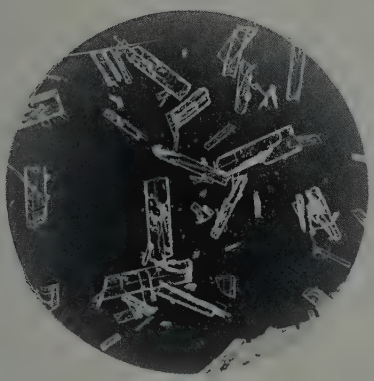


FIG. 3.

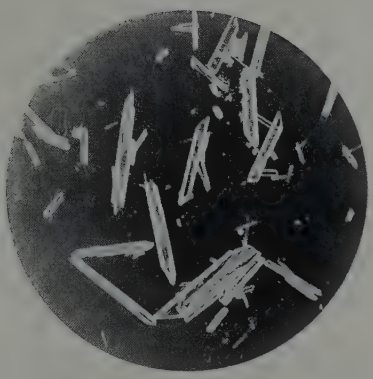


FIG. 4.

No. 9.

DERANGEMENT OF THE DIGESTIVE PROCESSES IN THE MILK-FED CALF DUE TO ABNORMAL CURD FORMATION IN THE FOURTH STOMACH.

By E. J. SHEEHY, B.Sc., F.R.C.Sc.I.,
University College, Dublin.

[Read May 29. Published separately July 19, 1934.]

Calves which are pail fed from infancy rarely make the same progress as those which are suckled by the cow. This is explained by the infrequency of feeding in the former case, and by the fact that, frequently, the milk when fed is not in the pure and fresh condition in which it leaves the cow. When feeding is done twice daily the quantity of milk ingested at each meal is six or eight times greater than that which would be taken at any one time by a calf which is at liberty to drink from the cow at frequent intervals.

In farm practice derangements of the digestive organs which are connected with the method of feeding very frequently occur during the period from birth to the age of about four weeks. Many deaths are caused thereby, and very often survivors develop into unthrifty individuals, which prove unprofitable. The financial loss, though difficult to estimate, runs into a very high figure.

Experimental (University College).

These disturbances of the alimentary system have been under investigation in the Animal Nutrition Department at Glasnevin for five years, each year a number of purchased calves, varying from 20 to 30, having been utilised for the purpose of the experiments. In addition, observations have been made during the past three years at farms in which these diseased conditions had previously caused considerable loss, and at which certain methods of prevention and treatment had, during the period of observation, been prescribed.

The investigation began in the 1928–29 rearing season. The calves had not received milk of any kind before arriving at the College, where they were treated as are most commercial animals throughout the country. They were fed twice daily with the mixed milk from five cows which had calved about four months previously. For the first few days they received 4 pints per day, for the remainder of the first week they got a daily allowance of 6 pints, which was increased first to 7 and then to 8 pints per day in the second week, subsequent to which each calf received 10 pints per day. The animals were comfortably housed in clean compartments, and fed in scrupulously clean

vessels. With this system of feeding about 50 per cent. of the calves developed symptoms of disorder to varying degrees of intensity, and, notwithstanding the application of a variety of popular treatments, five calves died before the disorder was brought under effective control, and a few of the survivors suffered severely from the check which they had received.

FIGURE 1.

The upper picture represents a calf suffering from digestive derangement due to abnormal curd formation in the 4th stomach.

The lower picture represents a healthy normal calf of the same age.

The development of the disordered condition was indicated by the following symptoms. At a particular meal the calf refused some of its allowance of milk: about the same time a noticeable dullness and want of vivacity became apparent; the ears dropped, and the back developed a slight arch: the animal remained in a lying position longer than the normal period, and had to be aroused to its food, and when lying it assumed a less easy attitude than the healthy animal. At one meal the milk would be consumed, only to be refused at the next, and so on till the condition became so bad that all milk was definitely refused. In the meantime the fæces changed from the normal semi-solid consistency to that of a fluid, and the normal yellowish tinge gave place to white: the calf had developed what is called white scour. The appearance of an animal suffering from such disorder in contrast with that of a healthy calf is shown in figure 1. Frequently there was evidence of unrest, and in some cases of considerable pain. Emaciation succeeded the diarrhœa, and unless relief was forthcoming death eventually occurred. Occasionally an animal, before the last stages were reached, would redevelop an appetite at a particular meal, and drink the full milk allowance, but such an individual died more readily than the less voracious one; in fact, it is the individual which, during all stages of the disorder, drank very sparingly which survived.

From the first appearance of evidence of digestive disorder among the calves various remedies contained in the literature on calf rearing and suggested by practical farmers were tried, and, though some acted as palliatives, none was found definitely effective either in the way of prevention or cure. Among these might be mentioned stimulants, sodium citrate, sodium carbonate, a combination of boiled starch, laudanum, and brandy (1), formalin, and boiled milk.

Post mortem examination[1] of the calves which died revealed no trace of disorganisation in any tissue other than the digestive organs, in which derangement was confined to the fourth stomach and upper portion of the

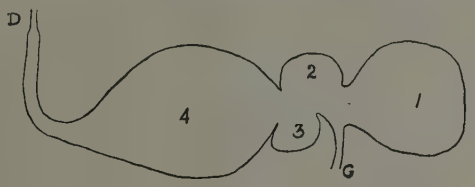


FIGURE 2.

Diagrammatic representation of the four compartments of the stomach of a calf at the age of two weeks.

G. = Gullet. D. = Duodenum.

duodenum. The relationship of the four stomach compartments to one another in a calf 14 days old is shown diagrammatically in figure 2. The wall of the

[1] The author is indebted to Prof. Craig, M.R.C.V.S., who kindly made a complete examination of the calves which died in the early stages of this investigation.

fourth stomach and upper duodenum was in all cases congested, and very noticeably so in parts, and the contents, which in some cases were discoloured by the blood from the ruptured blood-vessels of the stomach wall, included an abnormal amount of curd. This existed in the form of one large mass, usually accompanied by a number of smaller pieces, and the large clot which in some cases nearly filled the stomach had a consistency resembling that of cheese. Figure 3 shows the appearance of the opened stomach of a calf which died

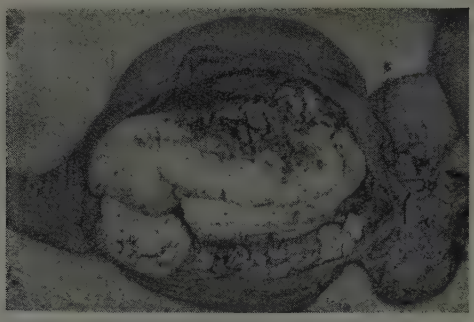


FIGURE 3.

Photograph of the 4th compartment of the stomach of a calf which died after showing the symptoms of ill-health, detailed in text. the lining has been slit and folded back so as to expose the abnormal collection of curd in the cavity.

after showing all the symptoms of disorder mentioned above. In a few cases the section of the curve showed concentric rings, which marked the accumulation of successive layers over a smaller central nucleus. In comparison with this the stomach of a healthy calf killed some hours after the taking of a normal feed of milk contained a soft and spongy curd, which easily breaks on handling. The stomach wall itself is of a delicate pink and uniform appearance.

The symptoms shown by the affected calves indicated that the seat of the trouble lay in the digestive tube, and the *post mortem* examinations located it in the fourth stomach. Notwithstanding the diarrhœa, the intestine was perfectly normal, and it was clear that the cause of the disordered condition was the inability of the stomach to deal with milk in the quantity and in the manner consumed. In the stomach, enzyme action first causes the milk to be curded (by rennin), the curd being subsequently digested into protein decomposition products (by pepsin). Calf stomach extract has a very strong rennin action, but a weak peptic action. The peptic enzyme requires a fairly acid medium in which to act, while rennin clotting takes place in a neutral or faintly acid solution. The consequence is that the curd of milk is effectively clotted in the stomach before any appreciable digestion of it by trypsin begins. Presumably, in the case of a calf which died as a result of the accumulation

of curd in the stomach, the clot formed from the milk taken at a feed was not completely digested before the next meal was taken. The residue acted as a nucleus, round which further curd formation developed, and with successive feeds of milk the residual curd became progressively larger and denser. The mucous lining of the fourth stomach of all suckling animals, including the calf, is very thin and delicate, and highly sensitive to unusually hard and persistent curd. The presence of the abnormal curd causes pain, which is exhibited by the symptoms of disorder described above. Continuous irritation of the stomach wall, through the operation of the gastro-iliac and gastro-colic nervous reflexes, causes an acceleration of the rate of peristalsis in the intestine to such a degree that diarrhœa is produced. Continued diarrhœa, because of the non-absorption of the intestinal contents, obviously causes the emaciation which always accompanies the final stages of the disorder. When the persistent curd assumes a certain density, due to continued pressure of the stomach wall thereon, the mucous lining becomes injured, congestion of the lining occurs, and in severe cases the blood-vessels are ruptured, and blood passes into the cavity of the stomach.

Attention was turned to the possibility of altering the curd formed in the stomach with the idea of rendering it more quickly digestible by the proteolytic enzyme (pepsin). A simple way of effecting this purpose suggested itself, namely, the dilution of the milk with water. When to a certain volume —X c.c.—of fresh milk, at the temperature of body heat, rennin is added, a clot is formed which has a volume equal to that of the original milk. If water is added to the milk, so that the volume becomes $1\frac{1}{3}$ X c.c., the clot formed on the addition of rennin is $1\frac{1}{3}$ X c.c. in volume; and if further dilution takes place, so as to bring the original volume of the milk and water mixture to $1\frac{1}{2}$ X c.c., the volume of the resulting curd is again $1\frac{1}{2}$ X c.c. The increase in the volume of clot formed from a definite quantity of milk obviously reduces its density. In fact, the clot becomes more and more open and spongy with increasing dilution, till a stage is reached when clot formation by rennin no longer takes place. The effect of dilution is illustrated in tubes 1, 2, and 3 of figure 4. Each tube was filled with milk up to the dark mark on the side, i.e. each contained the same volume of milk to begin with. To tube 2 some water was added, and to tube 3 a further quantity of water, and all three were clotted with rennin at 40° C. A clot appeared in each tube equal in volume to the entire volume of fluid therein: that in tube 2 being less dense than that in tube 1, and that in tube 3 still less dense. After some time the clot, in tests of this kind, begins to contract into a more compact mass, and to squeeze out a fluid around it, this process being more rapid in the case of the unadulterated milk in tube 1, in which for that reason the contents appear less homogeneous than in tubes 2 and 3. Presumably, the rennin clot of a mixture of milk and water is more readily disintegrated by a proteolytic enzyme than is the clot of unadulterated milk, and consequently the addition of water to the milk fed to the calf suggested itself as a remedy for the digestive dis-

organisation referred to. Not alone was it considered that the dilution of the milk would prevent the occurrence of disorder, but also that in the case of calves, in which the trouble had been precipitated, the withholding of milk for some days and the feeding of water, or a mixture of equal parts of milk and water, should constitute a suitable treatment.

Accordingly, two 12-day-old calves, which were seriously ill, and showing persistent diarrhœa, had their food altered to a mixture of 50 per cent. milk and 50 per cent. water.[2] This was fed four times per day, one pint being given

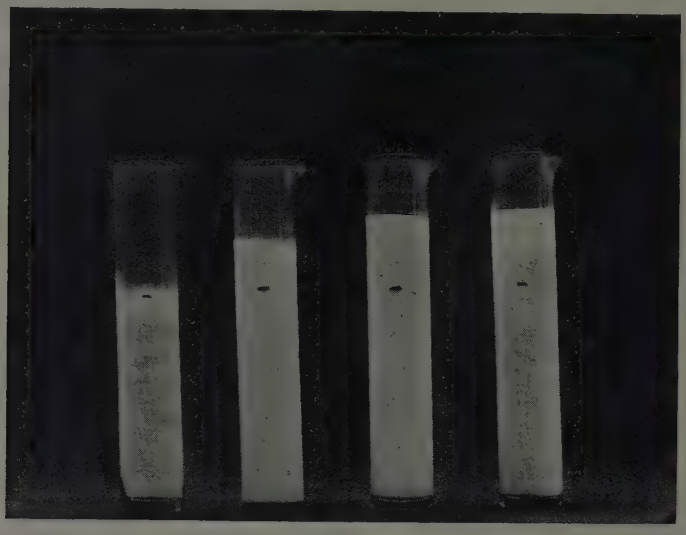


FIGURE 4.

Showing the comparative size (and consequently density) of the curd in 4 tubes, each containing the same quantity of milk—to the black stroke on glass. Tubes 2, 3, and 4 had water added, and in addition, tube 4 received a few drops of calcium chloride.

at each feed for the first day and 2 pints thereafter, till in 6 days the diarrhœa had disappeared, and the calf regained normal health. The recovery on this treatment was considered slow, and it was thought that a diet of water alone for a day or two would hasten recovery. With this idea in mind three other calves which later developed symptoms of disorder similar to the above received for one day water only, the quantity being two pints per feed given three times in the twenty-four hours. This was succeeded by milk and water in equal parts fed at the same rate for the next two days, by which time

[2] In all cases the water was boiled and then reduced to blood heat before adding to the milk. This treatment of the water is recommended for the purpose of precipitating any surplus lime.

the fæces had regained normal consistency, and the other symptoms of ill-health had disappeared. The quick return of normal health and activity in the case of the calves so treated was a very noticeable feature. Six calves which had been fed considerable quantities of unadulterated milk for the specific purpose of deranging the stomach function, and which developed all the symptoms of the disorder as anticipated, were given milk administered with a bottle for one day after the animals had refused to drink milk. By this time the calves remained continuously in a prostrate position, apparently unable to prolong life for any considerable period. In the case of two animals the administration of the milk was continued, and death occurred two days later, the *post mortem* examination revealing the usual congested stomach wall and a dense abnormal curd. From the other four animals milk was withheld after one day's bottle-feeding, and water was administered instead, in the same way and in like quantity. In twenty-four hours each of them began to show more interest in life, and in two days they were on their feet looking for food.

Before applying the adulteration treatment as a preventive the effect of feeding the unadulterated milk three times rather than twice daily to calves was tried. Somewhat less than two pints of unadulterated milk were given at each feed for the first two days of life, the allowance being increased to 2½ pints for the remainder of the first week, and 3 pints in the second. Notwithstanding the reduced quantity given at a single feed, digestive trouble occurred in quite a number of cases. To these the remedy already found successful was applied with complete satisfaction. However, it became clear that less trouble was experienced when the calves were fed three times daily rather than twice.

A batch of animals, twelve in number, were accordingly fed three times per day. All the milk which they consumed from the very beginning was diluted in the proportion of five of milk to one of water. Very much less trouble was experienced in the rearing of this group, the bill of health being much more satisfactory than anything experienced hitherto. Yet in a few instances individual animals went off their feed temporarily, and in two cases definite disorder with the accompanying diarrhœa developed. These were satisfactorily treated as in previous cases.

The recurrence of stomach disorganisation, even on diluted milk, did not, however, discount the value of dilution. It was taken as evidence of the mixture still containing excess of milk, at least in the case of certain individual calves whose digestive processes were apparently less vigorous than others. With the object of catering for all such animals the next batch of calves were fed, three times daily, on a mixture of three parts milk and one part water. The result was most gratifying, because they all maintained perfect health, and showed throughout the rearing season marked liveliness and activity, and an unfailing appetite for their food. The dull and listless air of so many of the calves previously fed on milk, and which was symptomatic of disturbance and pain in the stomach, was no longer to be seen, and the indifference at meal time was replaced by a boisterous anxiety.

Thereafter the dilution of the milk with water was adopted as a regular practice, and, though a considerable number of "bought in" calves have been reared in the Animal Nutrition Department of the College during the succeeding three seasons, no death, other than that experimentally caused, has occurred as a result of digestive disorganisation. Some disorder has occurred, however, but only in a few cases. These proved very interesting, in that they threw further light on the cause of weakness in what are commonly called delicate calves. In each case the milk diet was diluted till a mixture was prepared which did not precipitate trouble. One particular individual proved incapable of tolerating milk for the first three weeks of life till it was diluted with 50 per cent. of water, while still another sickened on milk until the dilution was increased to 25 per cent. of milk and 75 per cent. of water. Obviously the so-called delicate calves secrete a weak proteolytic stomach enzyme, which is capable of digesting the curd of milk only when the latter is highly diluted. The degree to which the dilution must be carried depends on the relative degree of weakness of stomach digestion.

It was further noted that by carefully observing the health of the calf the onset of digestive disturbance could be anticipated to some extent, so that by treatment its further development could be prevented. When two calves, 7 days old, fed on unadulterated milk in considerable quantity showed lack of appetite and developed listlessness, one was immediately put on a water diet for a day, succeeded by very much diluted milk for two days, when perfectly normal health had returned. The other was induced to continue its regular dieting, after two days of which the symptoms of disorder had reached the diarrhœa stage, by which time the derangement of the stomach had evidently reached a more acute condition. This test was repeated on three other pairs of calves with like results.

Some of the purchased animals in the latter years had been suckled by their mothers or had otherwise received "biestings" for one or two days after birth and before arrival at the farm. It was noted that such animals showed evidence of better health and more resistance to disease than those which from the beginning of life received milk from advanced lactation. It was noted also that, when, on a few occasions, "biestings" was available for feeding, a calf would tolerate in the unadulterated form a greater quantity of it than of the mixed milk of the herd. Accordingly, tests were arranged for the purpose of comparing the toleration of calves for unadulterated milk from freshly calved cows and from cows which were about half-way through their lactation period. Both groups were fed the same volume of milk twice daily, the quantity being that usually given to calves of similar age. All those fed the milk from freshly calved cows maintained good health, while a number of the other group showed symptoms of stomach derangement of varying stages of intensity, and serious consequences were averted only by treatment. Whenever opportunity arose observations were made on the toleration of calves for milk from freshly calved cows, and it became clear that almost invariably digestion was unaccompanied by any serious disorder.

During the investigation an opportunity also offered itself of observing the behaviour of calves fed on raw and on pasteurised milk. The milk in both cases was the mixed product from a herd or a number of herds, but the source of the raw and the pasteurised was different. Besides, the conditions of the test were not exactly comparable, in that the raw milk was fed in one year and the pasteurised in another. However, the observations were interesting. Out of 23 calves fed on raw milk unadulterated in one year of the investigation 11 became ill; of these 5 died, and a number of the survivors were weak for a considerable time. In another year, 16 calves were given pasteurised milk, unadulterated, in like quantity. Only 5 suffered from digestive trouble, and all but one survived the attack.

EXPERIMENTAL (PRIVATELY-OWNED FARMS).

The results of the earlier investigations herein reported were communicated to the Cow Testing Association members at a Congress held in January, 1931. To distinguish between contagious white scour in calves, caused by bacterial invasion of the body, and the scour or diarrhœa, resulting from the accumulation of abnormal curd in the stomach, the writer gave to the latter the name of nutritional white scour. Farmers who had suffered serious losses in the rearing of calves realised that in very many, if not in the majority of, cases the diarrhœa, which was one of the principal symptoms of the cases of ill-health among calves experienced on their farms, was of a nutritional source, and they decided to co-operate in trying the new treatment, namely, the dilution of the milk and the withholding of milk for 24 hours when symptoms of disorder presented themselves.

The experience on four farms is as follows:—On farm A, where some six or eight calves were lost each rearing season, the feeding of adulterated milk reduced the casualties from non-bacterial disorders to nil. Farm B was a dairy farm, where it had been found impossible to rear calves in previous years. On the new treatment there were no further casualties from white scour, and only 5 per cent. of the animals fed showed any signs of digestive trouble during rearing. In the case of farm C the owner, who was a most careful feeder, was worried by the occasional occurrence of sickness among his calves. The adoption of the new treatment enabled him to rear his calves without any of the setbacks previously experienced. Farm D harboured the bacteria of contagious white scour, to which was attributed the loss of a very considerable number of young calves each year. When, however, sick animals were treated as suggested above, and all animals had some water added to the milk from birth, the number of casualties was reduced to one-half, showing that in about half the cases of ill-health previously experienced among the calves the diarrhœa was of a nutritional rather than a bacterial origin. Reports from numerous other farmers were of a similar nature, and practical feeders are agreed that what was to them a very big problem has been solved.

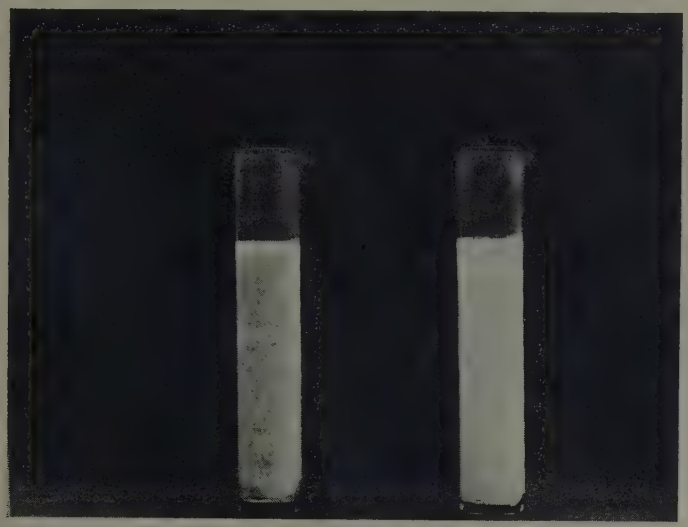
Discussion.

Under natural conditions of rearing, where newly-born mammals are suckled by the mother, digestive troubles are practically unknown. The artificial rearing of farm stock of any kind is, however, presented with certain difficulties. These are experienced in the feeding with cows' milk of orphan foals or bonhams (2). Difficulties of a similar nature occur in the feeding of the human infant with the milk of the cow. Indeed, one method of preparing cows' milk for the human infant includes the removal of part of the casein by rennin clotting. The milk of different species of animals varies in its response to rennin, i.e. the type of clot or curd produced. In the milk of one group of animals, to which the human, the horse, the ass, and pig belong, an open and inconsiderable clot is produced, while in that of another group, which includes the cow and the sheep, a very heavy and dense clot forms (3) (4). The consequence of feeding a foal or a young pig on unadulterated cows' or sheep's milk is, therefore, a much denser clot of milk in the stomach than would be produced by the same quantity of milk from its own species. The difference in the reaction of the milk of different species to rennin is largely a question of the relative proportion of globulin and casein in the milk, a relatively high proportion of the former inhibiting coagulation, while the latter favours it. In the case of the cow, the fluid which accumulates in the udder prior to calving, because of its relative richness in globulin, produces the same type of rennin clot as does the normal milk of the mare or ass. In colostrum or "biestings" the globulin (including the albumin) may run as high as 13 per cent. (3) (4). There is a progressive reduction till about the 12th day after parturition, when the percentage is down to less than a half per cent. Globulin, apart from the albumin, is very high in "biestings," and is very low in normal milk. This explains the readier digestibility of the curd formed from "biestings" or from the milk secreted during the first fortnight of lactation than from that subsequently yielded. Hence the greater success in the rearing of calves on the one than on the other, and the advisability of feeding calves as much as possible on the milk of freshly calved cows.

Figure 5 illustrates the effect of adding rennin, in the case of tube (*a*), to the milk from a cow 12 months calved, and, in tube (*b*), to milk from a cow 6 days after parturition. In the latter, the curd completely fills the tube, and its texture is open and spongy: in the former the clot has already begun to contract, and clear fluid may be seen not alone above the clot but between it and the glass container. Towards the end of lactation there is an increase in the proportion of casein in the milk, while the percentage of calcium also goes up.

It is not clear whether the increase in calcium towards the end of lactation is sufficient to influence the type of curd, but it is known that the alteration in the curd produced by adulterating milk with water is connected with the reduction in the percentage of calcium in the tube. The formation of rennin

curd occurs in two phases, namely, the chemical reaction and the clotting, for the latter part of which to take place effectively a certain concentration of calcium or other metallic cation is necessary (5) (6). Tube 4 of figure 4 brings out this point clearly. It was filled with milk to the same level as tubes 1, 2, and 3, namely, as far as the black mark on those tubes. To the milk of tube 4 was added about one-third its volume of water together with a few drops of calcium chloride. Rennet was added, as in the case of the other tubes, and all were incubated at 40° C. The clot in tube 4, as the picture shows, rapidly contracted, so that it became distinctly smaller in size, and, therefore, denser



(*a*) (*b*)

FIGURE 5.

Showing the comparative effect of rennin on the milk of a cow 12 months calved—tube (*a*), and the milk yielded by a cow 6 days after parturition—tube (*b*).

than that in tube 3. It did not, however, contract to the size of the clot in tube 1, which contains unadulterated milk. It will be noted that fluid is collecting in tube 4 as in tube 1 between the clot and the glass container. No doubt when water is added to milk there are other factors, such as the alteration of the concentration of the various ions in the fluid, which operate in the production of a more open, and hence more digestible, curd. At any rate digestion is rendered very much easier thereby, the curding properties of milk so adulterated being altered to resemble those of the normal milk of the horse, ass, and pig, or the milk of the cow previous to and for a very short time after parturition. The effect of adding water is equivalent to an increase in the concentration of globulin. While other methods of effecting the same purpose

may be suggested, the addition of water is, perhaps, the very simplest method which could be placed in the hands of the practical feeder of calves. When a curd accumulates in the stomach it is clear that there is only one agent which can dislodge it, namely, the activity of the proteolytic enzyme. The stomach loaded with curd already contains sufficient material to induce the flow of digestive juice, which, of course, requires water to float it into contact with the curd. Digestion of the curd in such circumstances will take place rapidly if an opportunity is given it, and the opportunity required is the withholding of all milk for a period of at least 24 hours, and the giving of very watery milk for a few days subsequent thereto.

Pasteurisation of milk lowers the tension of the curd formed by rennin, and apparently renders it easier to digest. It is well known that the boiling of milk decreases its calcium content by causing a precipitation of some calcium, and that boiled milk forms a more open clot with rennin than does the unboiled. In fact, the boiling of milk is frequently resorted to by farmers as a remedy in the case of diarrhœa or scouring in calves. Possibly pasteurisation may alter the condition of the calcium in milk in some way so as to bring about a change which, while not so pronounced as boiling, yet may attenuate the curding properties of the milk.

It has been noted that very severe cases of stomach disorder among milk-fed calves occur where stale milk is fed. This is probably due to the fact that a second clotting agent is introduced, namely, acid. An acid milk entering an acid stomach may in a very short time reach a point at which even in the absence of rennin clotting would occur, and in the meantime rennin clotting has been proceeding.

Practical feeders are aware of the more frequent occurrence of digestive derangement among purchased calves, particularly those procured from dealers, than in the case of calves born on the farm where rearing takes place. Though the transport and the exposure to chills predispose the animals to disease, yet the chief cause of trouble is the absence of "biestings" from the diet, and the fact that the calves are usually fed on the mixed milk of a herd, which, as shown above, produces a less easily digestible curd than milk from a cow at the beginning of lactation.

In addition to the difference in respect of the character of the rennin clot between the milks drawn from the cow at different stages of lactation there is a considerable variation as between individual cows in this respect (7), (8), (9). The difference is sufficient to be appreciable, at any rate, in the feeding of the human infant.

Summary.

1. The digestive derangements which occur in pail fed calves have been studied for five years, a total of 145 calves having been used in the experiments.

2. The symptoms are lack of appetite, lassitude, and diarrhœa (white scour). The disorder results in a severe setback to progress, and frequently ends fatally.

3. *Post mortem* examination located the seat of the disturbance in the fourth stomach, where the accumulation of dense, cheesy curd causes irritation and congestion of the stomach lining and sometimes rupture of blood-vessels, accompanied by a feeling of discomfort and pain.

4. The condition is precipitated by the inability of the proteolytic enzyme (pepsin) of the stomach of the affected calf to digest, in the interval between successive feeds, the clot produced by the action of rennin on the milk.

5. The dilution of the milk with water, with the object of favourably altering the character of the rennin clot, suggested itself as a simple method of treatment; and both as a preventive and a remedy it has proved highly successful, both at the College and on privately-owned farms.

6. The withholding of all milk and the feeding of water only for a day, followed by much-diluted milk for a few days, causes the disappearance of the accumulated curd from the stomach; for the purpose of definite prevention in all cases the proportion of water to be added to the milk diet depends on the number of times the calf is fed daily and on the relative vigour of the stomach proteolytic enzyme in the individual animal.

7. Calves fed on "biestings" and the milk from freshly calved cows in the unadulterated form showed practically no digestive trouble: the difference between the type of rennin curd formed by milk drawn at various stages of lactation and by the milk from different species of mammals is discussed.

8. Comparison is drawn between the type of curd produced by normal cow's milk to which water is added and that of the normal milk of the horse, ass, and pig, and the colostral milk of the cow.

9. Observations strongly indicated that pasteurised milk precipitates less stomach disorder than does raw milk fed in like quantity.

REFERENCES.

1. MINISTRY OF AGRICULTURE, NORTHERN IRELAND.—Leaflet No. 54.
2. GOLDING.—Pig Breeders' Annual (1928–29).
3. ASSOCIATES OF ROGERS.—Fundamentals of Dairy Science.
4. CH. PORCHER.—Le lait au point de vue colloidal.
5. HOLTER.—Biochem. Z., 255, 160 (1932).
6. MATTICK AND HALLETT.—Jr. Agr. Sc., 19, 452 (1929).
7. HILL.—Utah Agr. Exp. Sta. Bull. 207 (1928).
8. HILL.—Utah Agr. Exp. Sta. Bull. 227 (1931).
9. HILL.—Utah Agr. Exp. Sta. Circular No. 101.

No. 10.

THE APPROXIMATE DETERMINATION OF THE VIBRATION OF BEAMS AND THE WHIRLING OF SHAFTS.

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Certain problems in the theory of beams are complicated by the fact that some or all of the forces in action thereon depend as regards their magnitudes on the absolute, or on the relative, values of the deflections that occur at their points of application.

Thus in the vibration of a heavy beam under applied loads, effective forces are involved which depend on the accelerations at the neutral axis and thus upon the changes of its deflection curve from moment to moment. Moreover, damping forces depend on the velocities at the positions considered.

The introduction of such forces into the differential equation of moments renders the solution rather complicated.

In the whirling of shafts the centrifugal forces acting on the masses carried depend on the deflections at the loaded points, and the relative values of these deflections are involved in the determination of the whirling speeds.

The object of this paper is to indicate how the mathematical complexity may be greatly simplified by the introduction of a method of successive approximation, and be adapted further for drawing office calculation by the use of graphical or tabular methods.

1. The applicability of the method to the vibration of beams under the action of moving loads, steady or pulsating, (as in bridges) depends on the recognition of the fact that the accelerative and damping forces exercise a relatively small influence on the deflections compared with the gravitational loads.

Having regard to the small amplitude of the vibrations involved, and taking account of the time in which they are executed consequent on the motion of the moving or pulsating load, it can readily be seen that in practical problems the accelerations of the masses due to the vibratory motion are usually quite small compared with g, and thus to a first or rough approximation the accelerative forces may be neglected in comparison with the gravitational action of those masses.

The omission of these terms renders the solution of the differential equation quite simple and straightforward, and a first approximation to the true deflection curve at any instant is thus obtained. Thus the curves at several successive instants during the motion of the load may be plotted if desired.

A second approximation to the deflection curve may be obtained by including accelerative and damping forces in the equation of motion, the values of these

forces being derived by imagining the beam to execute vibrations in accordance with the deflection curves just obtained as a first approximation.

Obviously a third approximation may be obtained by utilising the accelerations and velocities derived from the second approximation to the deflection curves, and so on. Furthermore, various terminal conditions may be satisfied.

The author has given in an earlier[1] paper an account of the application of this method to certain practical problems in which algebraic solutions are presented.

In some of these, such as the vibrations consequent on the motion of a heavy mass along a uniform massive beam, the resulting mathematical expressions are somewhat lengthy. In this paper it will be shown how, as an alternative to algebraic treatment, the method of successive approximation may be applied in graphical or tabular form to such problems, thus leading, after about two approximations, to solutions sufficiently exact for all practical purposes (§§ 9–15).

Indeed, the uncertainty of the data renders inexact the results of calculation, however accurately carried out.

It is usual in the theoretical treatment to hypothesise that the ends of the beam are quite freely supported, that all the applied loads are quite definitely known and act at points on the neutral axis, that the elastic constants of the beam are specified, and that even if the beam is a built up or riveted structure it behaves as if it were uniform throughout its length as regards elastic properties and mass. In practice all these assumptions are subject to considerable uncertainty. Indeed if the terminal conditions approached a built-in state the effect on the deflection curve might be very serious, as will be realised by the observation that the central deflection of a simple uniform heavy beam supported freely is five times that of the same beam if it were built-in at both ends.

Accordingly in such practical problems the solution can only be an approximation to the true result, and, indeed, an approximate solution is all that is needed. Hence the application of a method of approximations to the solution of the equation of motion is fully justified.

2. Turning now to the whirling of shafts, the most common problem in practice is the determination of the first whirling speed of a shaft freely supported in bearings at its ends and carrying a number of massive loads. The evaluation becomes complex when the number of loads exceeds more than three or four. The method of successive approximation can, however, be employed to lead rapidly to a good approximation to the first whirling speed. This will be shown in the following pages algebraically, graphically, and tabularly (§§ 3–8).

Thus, in determining the first whirling speed of a shaft carrying a number of masses, the procedure is as follows:—

At first the centrifugal forces acting on the masses at the several positions

[1] Phil. Mag., Vol. VIII, July, 1929, pp. 66–97.

along the shaft and causing it to deflect from its static form are replaced by loads proportional to the masses simply, and the deflection curve corresponding thereto is determined.

For a second approximation the loads are taken proportional to the products of the several masses each multiplied by the corresponding deflection just found, and a new and more accurate displacement curve is obtained. From these new deflections a third approximation may be made if desired. Finally, the first whirling speed may be deduced as will be shown.

For the sake of simplicity and lucidity of explanation a very simple example has been chosen, viz. :—the determination of the whirling speeds of a light uniform shaft of negligible mass supported freely in bearings at its ends and carrying only two concentrated masses.

The method of treatment is, however, quite general and can include disk action, and its great advantage is exhibited when applied to complicated loads, and even to non-uniform beams.

According to the elastic theory of beams in motion or vibration, we regard the beam as the seat of certain elastic properties which at every instant are connected with the bending moment M and the deflection y at any position x in the length of the beam by the relation

$$M = \frac{EI}{\rho} \doteqdot EI\frac{d^2y}{dx^2}$$

where E is Young's Modulus, I is the moment of inertia of the cross-section of the beam about a line intersecting the neutral axis at right angles to the plane of bending, so that the product EI is the flexural rigidity of the beam, and ρ is the radius of curvature of the bent neutral axis at the position considered.

The bending moment M is caused by all the forces acting on the beam, viz. :—the applied forces, the effective or acceleration forces due to the motion of the mass of the beam itself or of any masses attached to it or moving with it, and also any forces due to damping.

The deflections are presumed to be small though not necessarily infinitesimal, and the stresses are supposed to lie within the elastic limit of the material.

3. We proceed now to determine the first whirling speed of a light uniform shaft of length l and flexural rigidity EI, supported freely at its ends and carrying heavy masses m_1 and m_2 concentrated at positions distant x_1 and x_2 from one end.

In particular we shall later suppose $m_1 = m_2 = m$ and $x_1 = \frac{1}{2}l$, $x_2 = \frac{3}{4}l$.

It may readily be shown that if a uniform shaft supported freely at each end be subjected to a number of loads P_s perpendicular to its length l, acting at points distant x_s $(= lz_s)$ from one end, the deflection y_r at any position $x_r(= lz_r)$ is given by the equation

$$\frac{6EI}{l^3} y_r = \sum_{s=0}^{s=r} P_s A'_{rs} + \sum_{s=r}^{s=l} P_s A_{rs}.$$

Where $$A'_{rs} = z_s(1 - z_r)(2z_r - z_r^2 - z_s^2)$$

and $$A_{rs} = z_r(1 - z_s)(2z_s - z_s^2 - z_r^2).$$

If the loads consist of the centrifugal forces acting on masses m_s placed at the several s positions, and ω is the speed of rotation, then

$$\frac{6EI}{\omega^2 l^3} y_r = \sum_{s=0}^{s=r} m_s A'_{rs} y_s + \sum_{s=r}^{s=l} m_s A_{rs} y_s.$$

Choosing the position r to coincide in turn with the several load, or s, positions we obtain a series of equations from which the displacements y_s may be eliminated and there results a determinantal equation in ω, the roots of which are the various whirling speeds, namely

$$\begin{vmatrix} A_{11} - \frac{\theta}{m_1} & A_{12} & A_{13} & . & . \\ A'_{21} & A_{22} - \frac{\theta}{m_2} & A_{23} & . & . \\ A'_{31} & A'_{32} & A_{33} - \frac{\theta}{m_3} & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{vmatrix} = 0$$

where $\theta = \dfrac{6EI}{\omega^2 l^3}$.

This method of solution was first given in an earlier[2] paper by the author, where its extension to include disk action is shown.

When there are only two loads P_1 and P_2 on a light uniform shaft the deflections at the load positions z_1 and z_2 are y_1 and y_2, given by

$$\frac{6EI}{l^3} y_1 = 2P_1 z_1^2 (1 - z_1)^2 + P_2 z_1 (1 - z_2)(2z_2 - z_2^2 - z_1^2)$$

$$\frac{6EI}{l^3} y_2 = P_1 z_1 (1 - z_2)(2z_2 - z_2^2 - z_1^2) + 2P_2 z_2^2 (1 - z_2)^2.$$

If $z_1 = \frac{1}{2}$ and $z_2 = \frac{3}{4}$ we find

$$\psi y_1 = 16P_1 + 11P_2,$$
$$\psi y_2 = 11P_1 + 9P_2,$$

where $$\psi = \frac{768\,EI}{l^3}.$$

If the masses at z_1 and z_2 are each equal to m, and if the forces are due to centrifugal action consequent upon the shaft rotating at speed ω in deflected form, we have

$$P_1 = m\omega^2 y_1, \quad P_2 = m\omega^2 y_2.$$

Hence if the shaft remains in dynamic equilibrium in the deflected position, we have

$$\phi y_1 = 16y_1 + 11y_2,$$
$$\phi y_2 = 11y_1 + 9y_2,$$

[2] *The Engineer*, November, 1909, paragraph 17, p. 491.

where $$\phi = \frac{768\,EI}{m\omega^2 l^3}.$$

Eliminating y_1 and y_2 we find ϕ, and thence the two values of ω^2 that satisfy the equations.

These give the two whirling speeds. They are the roots of the determinantal equation in ω,

$$\begin{vmatrix} \phi - 16 & 11 \\ 11 & \phi - 9 \end{vmatrix} = 0$$

or $$\phi^2 - 25\phi + 23 = 0,$$

or $$\phi = 24{\cdot}043 \text{ or } 0{\cdot}955,$$

so that $$\omega_1^2 = \frac{768\,EI}{24{\cdot}043\,ml^3}$$ and

$$\omega_2^2 = \frac{768\,EI}{0{\cdot}955\,ml^3}.$$

When the number of loads is small, as in this example, the determinantal equation is readily soluble, but in practice the number of loads to be taken into account is usually fairly large, and the solution of the determinantal equation is very troublesome.

In such cases the method of successive approximation now to be described may be used with great advantage to find the first whirling speed, or the first and second whirling speeds.

4. We next proceed by the method of successive approximation to find an approximate value for the first whirling speed in the example chosen.

In this case the shaft deflects in one loop with nodes at the bearings.

The deflections are caused by the centrifugal forces which, since the masses are equal, are themselves simply proportional to the deflections at the load positions.

As before, we have the equations

$$\psi y_1 = 16P_1 + 11P_2,$$
$$\psi y_2 = 11P_1 + 9P_2.$$

If now we know approximate values y_1' and y_2' for y_1 and y_2 we have approximately

$$\psi y_1' = 16P_1 + 11P_2,$$
$$\psi y_2' = 11P_1 + 9P_2,$$

Substituting for P_1 and P_2 the values $m\omega^2 y_1$ and $m\omega^2 y_2$ respectively we have approximately

$$\phi y_1' = 16y_1 + 11y_2,$$
$$\phi y_2' = 11y_1 + 9y_2,$$

Put $$\frac{y_2}{y_1} = \rho, \quad \frac{y_2'}{y_1'} = \rho', \text{ etc.}$$

$$\phi y_1' = y_1(16 + 11\rho),$$
$$\phi y_2' = y_1(11 + 9\rho),$$

Therefore $$\rho' = \frac{11 + 9\rho}{16 + 11\rho}.$$

Now as a first approximation let us suppose the forces proportional to the masses simply. This assumption of course does not do full justice to the mass at the position of greater deflection.

It is the relative values of the deflections at the loaded points that influence the magnitude of the whirling speeds and not the absolute values of the deflections. In other words the shape of the deflection curve is of importance rather than a general proportional increase or decrease of the magnitudes of the displacements, seeing that the two counteracting systems of forces, centrifugal and elastic, both increase or decrease in simple proportion to the general displacements, supposed small and within the elastic limit of the shaft.

At first, then, static gravitational conditions of deflection are assumed.

Hence $P_1 = P_2$, and we have $\rho' = \frac{11 + 9}{16 + 11} = 0{\cdot}741.$

Alternatively we might of course simply have started with $\rho = 1$ as a first assumption, leading to $\rho' = 0{\cdot}741$ as the corresponding value of ρ'.

Now for $\rho = 1$ we have $\phi y_1' = y_1(16 + 11)$ and making $y_1' = y_1$ (but not necessarily also y_2' equal to y_2) we have $\phi = 27$ as a first approximation. In like manner for $\rho' = {\cdot}741$ we have $\phi = 16 + 11\rho' = 24{\cdot}148$ as a second approximation.

For a better approximation we use the value of ρ', and thus obtain a closer value of ρ''.

Thus $$\rho'' = \frac{11 + 9\rho'}{16 + 11\rho'} = {\cdot}7316$$

and $$\phi = 16 + 11\rho'' = 24{\cdot}048.$$

Again $$\rho''' = \frac{11 + 9\rho''}{16 + 11\rho''} = {\cdot}7312$$

and $$\phi = 16 + 11\rho''' = 24{\cdot}043.$$

Thus we have started with the gravitational deflections as the values of y_1 and y_2, so that

$$\frac{\psi}{mg} y_1' = 16 + 11 = 27$$

$$\frac{\psi}{mg} y_2' = 11 + 9 = 20 \quad \text{leading to}$$

$$\rho' = \frac{y_2'}{y_1'} = {\cdot}741 \quad \text{as above.}$$

Accordingly, the procedure is as follows:—

We start with a likely and possible value of ρ, in the equation

$$\rho' = \frac{11 + 9\rho}{16 + 11\rho},$$

and we get in succession better values of ρ', *i.e.*, ρ'', ρ''', etc. Then $\phi = 16 + 11\rho'''$ gives closely the first whirling speed.

It will be noticed that $\phi = 24{\cdot}043$ agrees with the true value to three decimal places.

5. We shall next show how to find the second whirling speed in this example by a similar method of successive approximation.

At the second whirling speed the shaft is deflected into two loops, with nodes at the bearings and at one intermediate point.

The masses are on opposite sides of the line joining the bearings, and the centrifugal forces acting thereon are in opposite directions. The ratio ρ of the displacements is now negative.

Looking at the problem physically we see that if we substitute an approximate value of ρ for the true value we get from the equation a less approximate value. The values thus diverge from the true value instead of converging towards the true value as in the case of finding the first whirling speed.

Accordingly, to secure converging results we proceed with approximations in the opposite manner, and assume a likely and possible value for ρ', and use the equation to give the corresponding value of ρ.

Thus
$$\rho' = \frac{11 + 9\rho}{16 + 11\rho},$$

Therefore
$$-\rho = \frac{11 - 16\rho'}{9 - 11\rho'}.$$

To begin with we assume a possible value for ρ'. Let us take $\rho' = -1$. Then $\rho = -1{\cdot}35$. Taking this as the new value of ρ' we find $\rho = -1{\cdot}3669$. After a further similar step we find $\rho = -1{\cdot}3676$.

The corresponding value of ϕ is $16 + 11\rho$ or $0{\cdot}956$. This is very nearly the true value which is $0{\cdot}955$.

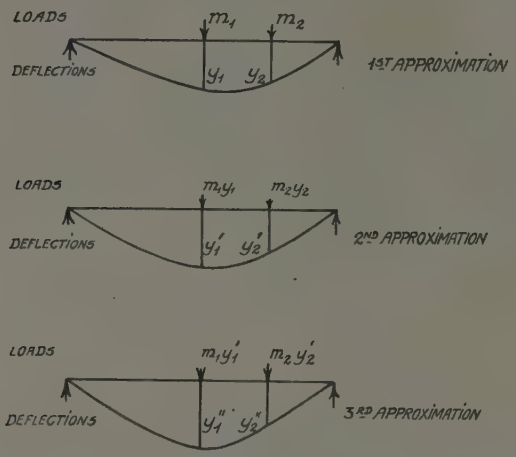
6. The solution of the problem with two equal masses is of course very simple. In practice the number of masses is usually fairly large, but the method of successive approximation may be applied in a straightforward manner.

In finding the first whirling speed we commence by obtaining the gravitational deflection curve in which the forces acting on the masses are proportional to the masses. We then proceed to find the deflection curve of the shaft when the load system consists of the masses each multiplied by the corresponding gravitational displacement. These products are regarded as the new system of forces, and the resulting deflection curve is determined.

As a further approximation these new deflections are each multiplied by the corresponding mass to give a revised set of forces for which a revised deflection curve is obtained.

The consecutive steps may be illustrated diagrammatically as in fig. 1.

Fig. 1.



This work may be carried out analytically, but it can be handled very conveniently by graphical or tabular methods. We proceed to indicate the fundamental features of these processes.

7. We commence with the tabular method of determining the first whirling speed in the example already chosen, viz.—two equal masses placed at distances $\frac{1}{2}l$ and $\frac{3}{4}l$ from one end on a light uniform shaft supported freely in bearings at its ends.

Let us suppose that the length of the shaft is divided into 8 equal parts, and let us number the points of division 0, 1, 2, ... 7, 8. We first assume that the loads W are proportional to the masses on the shaft, and from this system of forces we find the bending moments M at each of the points of division.

Then since $$EI\frac{d^2y}{dx^2} = M$$

we have $$EI\frac{dy}{dx} = \int M\,dx,$$

so that the change in the slope $\frac{dy}{dx}$ between adjacent positions x_1 and x_2 is equal to

$$\frac{1}{EI}\int_{x_1}^{x_2} M\,dx.$$

Now the definite integral is equal to the area of the M curve between the ordinates x_1 and x_2, or approximately to $\frac{1}{2}(M_1 + M_2)\delta x$, if δx is the common interval $x_2 - x_1$. Thus step by step we construct the curve for $\frac{dy}{dx}$ versus x.

In like manner

$$y_2 - y_1 = \int_{x_1}^{x_2} \frac{dy}{dx} dx,$$

and this is approximately equal to

$$\tfrac{1}{2}\left[\left(\frac{dy}{dx}\right)_1 + \left(\frac{dy}{dx}\right)_2\right] \delta x.$$

Accordingly we have an approximation to the values of the deflection at the various points considered along the shaft.

The operations can be arranged tabularly as follows:—

TABLE I.

To find the reactions at the ends (R_1 and R_2).

Load position.	Load.	Reaction.	Moments of loads.	Moments of reactions.
0	—	R_1	—	0
4	W	—	$4W$	—
6	W	—	$6W$	—
8	—	R_2	—	$8R_2$
	Sum $= 2W = R_1 + R_2$		Sum $= 10W = 8R_2$	
			$R_2 = 1\frac{1}{4}W$	
			$R_1 = \frac{3}{4}W$	

The difference between consecutive values of the bending moment

$$M_{n+1} - M_n = (R_1 - \Sigma_0^n W)\,\delta x,$$

if δx is the common interval of length. Hence

TABLE II.

To find the bending moments.

Position x	$\frac{4 \text{ Reaction}}{W}$	$\frac{4 \text{ Load}}{W}$	$\frac{4}{W}\Sigma$ (load)	$\frac{4}{W}(R_1 - \Sigma W) = \frac{4\delta M}{W}$	$\frac{4M}{W}$
0	3	0	0	3	0
1	—	0	0	3	3
2	—	0	0	3	6
3	—	0	0	3	9
4	—	4	4	− 1	12
5	—	0	4	− 1	11
6	—	4	8	− 5	10
7	—	0	8	− 5	5
8	5	0	8	− 5	0

The figure in the nth row of column 6 is obtained by adding together the figures in the $(n-1)$th row of both columns 5 and 6.

TABLE III.

To find the deflections.

Position x	$\frac{4}{W}M$	$\frac{8}{W}\int Mdx$	$\frac{16}{W}\iint (Mdx)\,dx$	$\frac{x}{8}\times$ final value	$\frac{16EI}{W}$. deflection.
0	0	0	0	0	0
1	3	3	3	104	101
2	6	12	18	208	190
3	9	27	57	312	255
4	12	48	132	416	284
5	11	71	251	520	269
6	10	92	414	624	210
7	5	107	613	728	115
8	0	112	832	832	0

Here the figure in the nth row of column 3 is obtained by adding together the figures in the $(n-1)$th row of column 3 and those in the nth and $(n-1)$th rows of column 2.

In like manner column 4 is derived from column 3.

Finally the bottom figure in column 4 is divided by the number of divisions used (8), and x times this quotient in column 5 less the figure in row x of column 4 gives the figure in row x of column 6.

The deflection curve thus obtained in column 6 is an approximation in form to the gravitational curve consequent on two equal forces W at $\frac{1}{2}l$ and $\frac{3}{4}l$.

We now proceed to a second approximation to the centrifugal deflection curve. Here we choose two loads proportional to the masses multiplied by the approximate deflections just found at the load positions $\frac{1}{2}l$ and $\frac{3}{4}l$. It is convenient to retain whole numbers in these tables, and the scale can be fixed afterwards if required. In whirling speed problems only the relative values of the deflections are needed.

TABLE Ia.

Load position.	Load.	Reaction R	Moments of loads.	Moments of reactions
0	—	R_1	—	0
4	$284W$	—	$1136W$	—
6	$210W$	—	$1260W$	—
8	—	R_2	—	$8R_2$

$494W = R_1 + R_2$ $\qquad$ $2396W = 8R_2$

$R_2 = 299{\cdot}5W$

$R_1 = 194{\cdot}5W$

TABLE IIA.

Position x	$\frac{2 \text{ Reaction}}{W}$	$\frac{2 \text{ load}}{W}$	$\frac{2}{W}\Sigma(\text{load})$	$\frac{2}{W}(R_1 - \Sigma \text{load}) = \frac{2}{W}\delta M$	$\frac{2M}{W}$
0	389	0	0	389	0
1	—	0	0	389	389
2	—	0	0	389	778
3	—	0	0	389	1167
4	—	568	568	− 179	1556
5	—	0	568	− 179	1377
6	—	420	988	− 599	1198
7	—	0	988	− 599	599
8	599	0	988	− 599	0

TABLE IIIA.

Position x	$\frac{2}{W}M$	$\frac{4}{W}\int M dx$	$\frac{8}{W}\iint (M dx)\, dx$	$\frac{x}{8} \times$ final value.	$\frac{8EI}{W}$ deflection.
0	0	0	0	0	0
1	389	389	389	13288	12899
2	778	1556	2334	26576	24242
3	1167	3501	7391	39864	32473
4	1556	6224	17116	53152	36036
5	1377	9157	32497	66440	33943
6	1198	11732	53386	79728	26342
7	599	13529	78647	93016	14369
8	0	14128	106304	106304	0

Now $$\phi y_1 = m_1 A_{11} y_1 + m_2 A_{12} y_2 + m_3 A_{13} y_3 + \ldots\ldots,$$
where A_{11}, A_{12}, etc. are known functions of the position of the loads (§ 3).

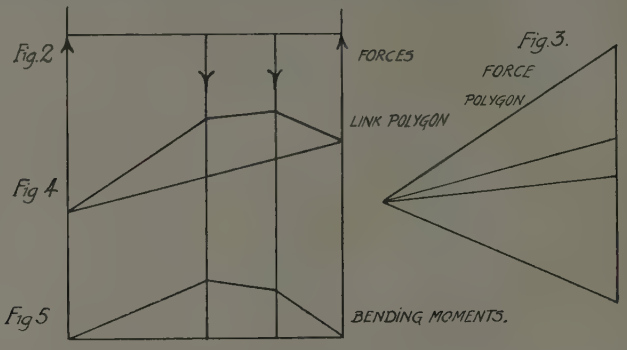
Substituting for y_1, y_2, etc., the approximate values obtained we find ϕ and thence ω the first whirling speed.

Here
$$\phi \times 36036 = 16 \times 36036 + 11 \times 26342.$$
Therefore $\phi = 24{\cdot}041$.

Had we stopped at the first approximation we would have had
$$\phi \times 284 = 16 \times 284 + 11 \times 210.$$
Therefore $\phi = 24{\cdot}133$.

8. There are two alternative graphical processes, one based on the form
$$M = EI\frac{d^2y}{dx^2} \quad \text{and the other on} \quad M = \frac{EI}{\rho}.$$

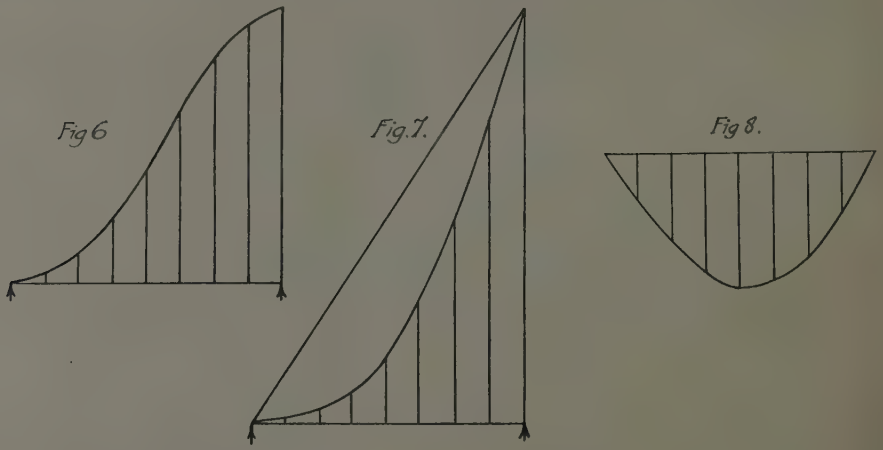
In both cases we first obtain by the well-known graphical method the bending moment curve for the system of loads under consideration.



The forces acting on the beam in fig. 2 are drawn to scale in turn on the vertical line of fig. 3 and the points of division joined to a convenient pole O. In fig. 4 lines are drawn in sequence parallel respectively to the corresponding lines through O; and the end points joined. The parallel through O to this closing line of the link polygon divides the vertical in fig. 3 into the reactions at the ends of the beam. Then, as is well known, the vertical intercepts on the link polygon represent the bending moments at the corresponding positions on the beam and are plotted in the bending moment diagram fig. 5.

Proceeding now by the first alternative method we divide the length of the beam into a number of equal parts and erect ordinates on the bending moment diagram. We construct a new diagram fig. 6 with the beam as base divided into the same number of equal parts as the bending moment diagram fig. 5.

The first ordinate of fig. 6 is made equal to the ordinate of fig. 5 at the middle of the first interval. The second ordinate of fig. 6 is equal to its first ordinate plus the middle ordinate of the second interval of fig. 5. The third ordinate of fig. 6 exceeds the second ordinate by the middle ordinate of the third interval of fig. 5; and so on.



From the method of construction it will be seen that the ordinates of fig. 6 represent approximately to some scale

$$EI\frac{dy}{dx} \quad \text{which is equal to} \quad \int_0^x M\,dx.$$

Fig. 7 is obtained from fig. 6 exactly as fig. 6 was obtained from fig. 5.

Fig. 7 approximately represents the deflection curve y, since

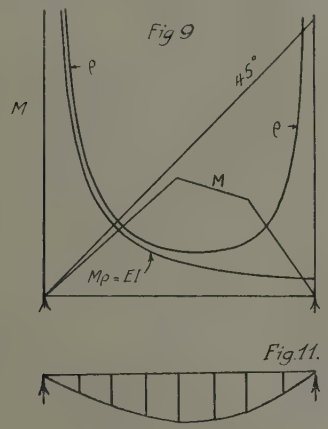
$$EI\,y = \int_0^x \left(EI\frac{dy}{dx}\right) dx.$$

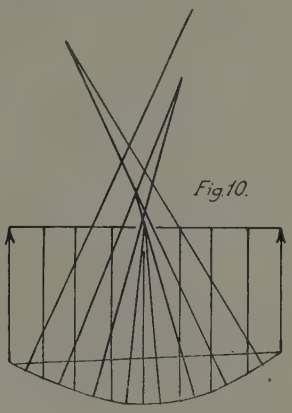
Finally the extreme points of fig. 7 are joined by a straight line, and the deflections are proportional to the intercepts on the ordinates between this line and the curve, and are plotted in fig. 8.

Thus the deflection curve has been approximately obtained.

Alternatively, using the form $M = \frac{EI}{\rho}$ we can construct the deflection curve from the bending moment diagram fig. 5. We first obtain a curve for ρ at each point of the diagram.

In fig. 9 we plot M from fig. 5, and also the equilateral hyperbola $M\rho = EI$ as shown. Drawing a line at 45° through the origin we readily obtain the curve for ρ. Thus, for any value of x proceed vertically to meet the M curve, then horizontally to meet the equilateral hyperbola, where the abscissa is ρ, then vertically to the 45° line, where the ordinate also is ρ, and then horizontally to the ordinate through x, which gives the value of ρ at x. In this way the curve giving ρ at each point of the beam is rapidly obtained.





Finally in fig. 10 the deflection curve is obtained as follows:—

Divide the length of the beam into a number of equal parts. Commence at some convenient point, for example, the central position, where ρ is, say ρ_0, and choosing any convenient point as centre draw an arc of a circle of radius proportional to ρ_0 on a convenient scale extending to half an interval on each

side of that position. Join one end of the arc to the centre and in this line take a new centre distant from the arc by ρ_1, the value on the chosen scale of ρ corresponding to the next interval. With this centre and radius ρ_1 draw a further circular arc to the middle of the adjacent interval. Thence determine a new centre corresponding to ρ_2 the next value of ρ. And so on. Thus the curvature of the beam is determined, and the deflections may be plotted therefrom to a horizontal base in fig. 11.

At each point the curve approximately satisfies the equation

$$\rho = \frac{EI}{M}$$

to the appropriate scale.

Having obtained approximately by graphics the curve of deflection of the beam under the tentative system of loads, we proceed to a better approximation by choosing a new system of loads dependent on the deflections just found. In the example of whirling speed the new loads are proportional to the masses multiplied by the corresponding deflections.

The graphical process is then repeated, the new bending moment diagram is prepared, and a second approximation to the deflection curve is obtained. If necessary a third approximation can be made.

Both the tabular and graphical methods are straightforward and simple in application, and are especially useful when the number of loads on the shaft is large.

9. We now proceed to show how the methods just described may be employed in the solution of the vibration of beams subjected to moving loads and moving forces, steady or pulsating, including damping.

Already the author has[3] given algebraic solutions based on the method of successive approximation. Accordingly, it is only necessary here to indicate how the tabular and graphical processes may be applied.

A simple example will suffice for illustration. A uniform massive beam rests horizontally on supports at its ends, and is subjected to a vertical force which moves along the beam with uniform velocity from one end to the other. It is required to find the deflection curve from time to time throughout the motion.

The cross-sectional dimensions of the beam are supposed to be small, so that the angular acceleration of sectional pieces may be regarded as negligible.

The forces on the beam include the applied force, the weight of the beam itself, the effective forces due to the acceleration of motion of the particles of the beam, and the damping forces depending on the velocity of the various particles.

Now the accelerative and damping forces are relatively small, so we omit them in a first approximation. We then obtain by tabular or graphical construction, in the manner described above, the deflection curves corresponding

[3] Phil. Mag., Vol. VIII, July, 1929.

to each of a number of equidistant positions of the moving load as it traverses the beam.

We can now plot the deflection to a time base for each position on the beam, and from this curve, showing the time variation of deflection, we obtain the slope or velocity at several instants of time.

By plotting the velocity against the time for any chosen point on the beam we get the accelerations of that point from the slopes corresponding to several instants during the motion.

Accordingly, we have a first approximation to the velocity and acceleration at each point corresponding to different times, or to different positions of the moving force as it traverses the beam.

We are now in a position to seek a second approximation as follows :—

Corresponding to each of the positions of the moving force that have been studied we know approximately the values of the velocity and acceleration at each point of the beam. We use these values to obtain the damping and acceleration terms which we now include with the other forces. In this way we obtain better deflection curves corresponding to the chosen positions of the moving force. These curves may be used to give revised values of the velocity and acceleration and further approximations may be obtained by a repetition of the process.

The graphical constructions for the velocity and acceleration at each position on the shaft can readily be carried out.

Corresponding to the position chosen for examination we plot to a time base the values of the deflection there, the total time being that required for the moving force to traverse the beam.

Divide the base of this curve of deflection plotted against time into a number of equal intervals, and erect ordinates.

At the middle of each interval erect an ordinate equal or proportional to the difference between the ordinates at each side of that interval. The curve connecting the ends of these ordinates represents approximately the velocities of the chosen point on the beam.

The acceleration curve is obtained from the velocity curve in exactly the same way as the velocity curve was obtained from that of the deflection.

The work can be kept fairly compact, particularly in the tabular solution, as will be seen.

10. Let it be required to find by the tabular method the deflections when the moving force is P, the mass of the beam ml, damping being included. We desire to find a particular integral of the equation of motion.

Let us suppose the length of the beam divided into say 8 equal parts, the mass of each part being $ml/8$. Preferably a larger number of divisions should be employed, but 8 is convenient for illustration.

The weight of the beam causes a deflection y given by

$$24EIy = mgl^4z(z^3 - 2z^2 + 1).$$

This deflection is to be added to that due to the force P. The latter deflection varies at every point with the motion of P, and accordingly it is the changes of deflection at any point due to this item that constitute the motion of the beam, and from them the velocity and acceleration are derived.

For a first approximation we omit the acceleration and damping terms, and find the deflections at the various points along the beam consequent on the motion of the force P.

Firstly, with the load P in position 1 we find the deflections at the positions 1, 2, 3, 4, 5, 6, 7. This may be done by finding the bending moments due to P acting in this position, and thence by tabular method, as in paragraph 7 above, we obtain the deflections. This must be repeated for P in the six other positions 2, ... 7.

It is, however, more expeditious to find the deflections by formula in this first stage for the single force P.

As in paragraph 3 the deflection y at $x = lz$ due to P at $x_1 = lz_1$ is, when $z < z_1$,

$$\phi y = z(1 - z_1)[1 - z^2 - (1 - z_1)^2];$$

and when $z' > z_1$,

$$\phi y' = z_1(1 - z')[1 - z_1^2 - (1 - z')^2].$$

where $\phi = \dfrac{6EI}{Pl^3}$.

We can then readily fill in the following table of deflections :—

TABLE IV.

Values of $4 \times 8^3 \times \phi y$.

x_1 =	0	1	2	3	4	5	6	7	8
x = 0	0	0	0	0	0	0	0	0	0
1	0	49	81	95	94	81	59	31	0
2	0	81	144	175	176	153	112	59	0
3	0	95	175	225	234	207	153	81	0
4	0	94	176	234	256	234	176	94	0
5	0	81	153	207	234	225	175	95	0
6	0	59	112	153	176	175	144	81	0
7	0	31	59	81	94	95	81	49	0
8	0	0	0	0	0	0	0	0	0

It will be observed that, as may be expected from the symmetry of the load positions about the centre, the table is symmetrical about both diagonals so that the labour of calculation can be reduced to about one third.

The table gives the deflections at each of 9 positions corresponding to each of 9 positions of the moving load.

By subtracting, one from the other, adjacent values of the deflection in the horizontal rows we get figures approximately proportional to the vertical velocities.

TABLE V.

Velocities.

x_1 =	0	1	2	3	4	5	6	7	8
x = 0	0	0	0	0	0	0	0	0	0
1	49	32	14	− 1	− 13	− 22	− 28	− 31	
2	81	63	31	1	− 23	− 41	− 53	− 59	
3	95	80	50	9	− 27	− 54	− 72	− 81	
4	94	82	58	22	− 22	− 58	− 82	− 94	
5	81	72	54	27	− 9	− 50	− 80	− 95	
6	59	53	41	23	− 1	− 31	− 63	− 81	
7	31	28	22	13	1	− 14	− 32	− 49	
8	0	0	0	0	0	0	0	0	

Thus we have, at each value of x, the values of the velocities as the moving load traverses the beam through the several positions. The values of the velocity corresponding to the times when the load is at the several positions $x_1 = 1, 2, \ldots . 7$, may be obtained from the table by interpolation.

From Table V we deduce the vertical accelerations by subtracting adjacent values in the rows from one another.

TABLE VI.

Accelerations × (− 1).

x_1 =	0	1	2	3	4	5	6	7	8
x = 0	0	0	0	0	0	0	0	0	0
1	− 49	17	18	15	12	9	6	3	− 31
2	− 81	18	32	30	24	18	12	6	− 59
3	− 95	15	30	41	36	27	18	9	− 81
4	− 94	12	24	36	44	36	24	12	− 94
5	− 81	9	18	27	36	41	30	15	− 95
6	− 59	6	12	18	24	30	32	18	− 81
7	− 31	3	6	9	12	15	18	17	− 49
8	0	0	0	0	0	0	0	0	0

11. The reversed effective forces are equal to the accelerations each multiplied by the corresponding mass, which in this case is the same from one division to the next.

The forces due to damping are equal to the products of the damping coefficient and the velocities obtained from Table V.

Now all the displacements are supposed small, and the total deflection at any point due to all the loads is the sum of the separate deflections due to each load separately.

We already know the deflection curve due to the heavy shaft under its weight alone, and the curves due to the load P in its 7 positions.

To these deflections we have to add algebraically those due to damping and acceleration.

The damping forces at the middle of each interval are proportional to the figures in Table V, and from them by interpolation arithmetically, or more accurately from a curve drawn through the several points, we obtain the forces corresponding to each value of x_1. We shall fix the various scales of the tables later.

For each value of x_1 or of the time we then have a set of damping forces acting at the several positions x proportional to the figures in the corresponding column. Thence we find the resulting bending moments, which may be obtained as in Tables I and II and the deflections as in Table III. These bending moments are given in Table VII.

TABLE VII.

Bending moments due to the damping forces are proportional to—

$x_1 =$	0	1	2	3	4	5	6	7	8
$x = 0$	0	0	0	0	0	0	0	0	0
1	0	469	339	163	− 26	− 201	− 341	− 431	0
2	0	857	632	313	− 38	− 367	− 632	− 803	0
3	0	1101	831	431	− 28	− 469	− 829	− 1063	0
4	0	1170	900	490	0	− 490	− 900	− 1170	0
5	0	1063	829	469	28	− 431	− 831	− 1101	0
6	0	803	632	367	38	− 313	− 632	− 857	0
7	0	431	341	201	26	− 163	− 339	− 469	0
8	0	0	0	0	0	0	0	0	0

The figures in this table show an expected symmetry about the centre. From these bending moments we now find as in Table VIII the deflections (as done in Table III) for each value of x_1.

TABLE VIII.

For $x_1 = 1$.

	M	$2\int Mdx.$	$4\iint (Mdx)\,dx.$	$\frac{x}{8} \times$ final value.	$4Ely$
$x = 0$	0	0	0	0	0
1	469	469	469	11918	11449
2	857	1795	2733	23836	21103
3	1101	3753	8281	35754	27473
4	1170	6024	18058	47672	29614
5	1063	8257	32339	59590	27251
6	803	10123	50719	71508	20789
7	431	11357	72199	83426	11227
8	0	11788	95344	95344	0

Similar tables are constructed for $x_1 = 2, 3, 4, \ldots . 7$, and thence we obtain a table of the deflections due to the damping forces as follows:—

TABLE IX.

$x_1 =$	0	1	2	3	4	5	6	7	8
$x = 0$	0	0	0	0	0	0	0	0	0
1	0	11449	8667	4575	− 156	− 4797	− 8669	− 11227	0
2	0	21103	16024	8511	− 222	− 8825	− 16024	− 20789	0
3	0	27473	20947	11227	− 158	− 11449	− 20945	− 27251	0
4	0	29614	22676	12278	0	− 12278	− 22676	− 29614	0
5	0	27251	20945	11449	158	− 11227	− 20947	− 27473	0
6	0	20789	16024	8825	222	− 8511	− 16024	− 21103	0
7	0	11227	8669	4797	156	− 4575	− 8667	− 11449	0
8	0	0	0	0	0	0	0	0	0

The symmetry of the figures about the centre of the table will be noticed.

In like manner we obtain the deflections due to the accelerative forces which are proportional to the figures in Table VI.

We first find the bending moments due to these forces corresponding to each position x_1 of the load.

Thus for $x_1 = 1$ we have from Table VI the accelerative forces F in column 2 of Table X.

TABLE X.

x	Accelerative force F.	$(8-x)F$	$R_1 - \Sigma F$	M
0	0	0	49	0
1	17	119	32	49
2	18	108	14	81
3	15	75	− 1	95
4	12	48	− 13	94
5	9	27	− 22	81
6	6	12	− 28	59
7	3	3	− 31	31
8	0	0		0
		8) 392		
		R_1 = 49		

Multiplying the values of F by 7, 6, 5 . . . . respectively we obtain column 3, and the sum of this column is 8 times the reaction at the end $x = 0$, so that $R_1 = 49$.

In column 4 $R_1 - \Sigma F$ is entered, in which the entry in row n is obtained by subtracting from the entry in row $n-1$ the value of F in row n of column 2. Then in column 5 for M the entry in row n is got by adding to that in row $n-1$ the entry in row $n-1$ of column 4. The procedure is expeditious.

The bending moments are thus found to be proportional to the figures in Table IV.

Then for each value of x_1 we obtain the corresponding deflections as in Table XI.

TABLE XI.

For $x_1 = 1$.

	M	$2\int Mdx$	$4\int\int(Mdx)dx$	$\frac{x}{8}$ × final value.	$4EIy$.
$x = 0$	0	0	0	0	0
1	49	49	49	1036	987
2	81	179	277	2072	1795
3	95	355	811	3108	2297
4	94	544	1710	4144	2434
5	81	719	2973	5180	2207
6	59	859	4551	6216	1665
7	31	949	6359	7252	893
8	0	980	8288	8288	0

Obtaining similar tables for $x_1 = 2, \ldots . 7$ we collect the deflections due to the accelerative forces in Table XII.

TABLE XII.

Accelerative deflections proportional to—

x_1 =	0	1	2	3	4	5	6	7	8
x = 0	0	0	0	0	0	0	0	0	0
1	0	987	1795	2297	2434	2207	1665	893	0
2	0	1795	3284	4229	4504	4099	3100	1665	0
3	0	2297	4229	5491	5894	5397	4099	2207	0
4	0	2434	4504	5894	6384	5894	4504	2434	0
5	0	2207	4099	5397	5894	5491	4229	2297	0
6	0	1665	3100	4099	4504	4229	3284	1795	0
7	0	893	1665	2207	2434	2297	1795	987	0
8	0	0	0	0	0	0	0	0	0

There is symmetry about both diagonals.

Had we divided the beam into 16 equal parts instead of 8, the central deflection figure would have been 6511 instead of 6384. The more exact figure obtained analytically is 6554.

12. We now fix the scales of the figures in the various tables. In some cases we have introduced numerical factors to avoid fractions. Let T stand for tabular figure, v the uniform velocity of P along the beam of length l, and b the coefficient of damping. Then we have

Item.	Table No.	Value.
Load	—	P
Deflection due to P	IV	$\frac{TPl^3}{3 \times 8^4 \times EI}$
Vertical Velocity	V	$\frac{TPl^2 v}{3 \times 8^3 \times EI}$
Vertical Acceleration	VI	$\frac{TPl v^2}{3 \times 8^2 \times EI}$
Damping force	V	$\frac{TbPl^2 v}{3 \times 8^3 \times EI}$
Accelerative force	VI	$\frac{TmPl^2 v^2}{3 \times 8^3 \times EI}$
B.M. due to damping forces	VII	$\frac{TbPl^3 v}{6 \times 8^4 \times EI}$
B.M. due to accelerative forces	IV	$\frac{TmPl^3 v^2}{3 \times 8^4 \times EI}$
Deflection due to damping forces	IX	$\frac{TbPl^5 v}{3 \times 8^7 \times (EI)^2}$
Deflection due to accelerative forces	XII	$\frac{TmPl^5 v^2}{12 \times 8^6 \times (EI)^2}$

Knowing the scales of the several tables we can add the deflections appropriately to obtain the total deflection due to all the actions jointly.

For a numerical example let it be given that $mlg = \frac{2}{5}P$. Further let $v = 60$ ft. per sec. and $l = 100$ feet, and let the central deflection due to P be $\frac{1}{480}$ of the span, so that $EI = 10\,Pl^3$. Also let it be given that $b = \frac{3}{16}ml$.

We can now add the tabular deflections. For example, the deflection at quarter span ($x = 2$) when the force is there ($x_1 = 2$)

due to P is $\quad 144 \times \dfrac{Pl^3}{3 \times 8^4 \times EI} = 1{\cdot}406$ inches;

due to the weight of the beam is $\quad 0{\cdot}445$ inch;

due to damping is $\quad 16024 \times \dfrac{bPl^5v}{3 \times 8^7 \times (EI)^2} = 0{\cdot}0043$ inch;

due to acceleration is $\quad 3284 \times \dfrac{mPl^5v^2}{12 \times 8^6 \times (EI)^2} = 0{\cdot}0056$ inch.

Thus in the example it is seen that at this position the effect on the deflection of damping and acceleration is small, and a further approximation is unnecessary.

13. Another example of the tabular process will now be given.

A mass M moves with uniform velocity along a uniform beam of mass m per unit length which is freely supported horizontally at its ends. It is required to find the deflection curves, omitting damping.

Here the total deflection is made up of several parts:—

(a) Deflection due to weight of beam alone.

(b) Deflections due to weight of the moving load M in several positions x_1.

(c) From the sum of (a) and (b) we can find approximately the velocity and acceleration of the mass M as it passes through each point x_1, and thence the accelerative force on M. Hence we find the bending moment and deflection due to the acceleration of M.

(d) From (b) we get the velocity and acceleration of each point x; thence the accelerative force on m at each point x.

Hence we find the bending moment and deflection due to the acceleration of m.

Adding algebraically the deflections in (a), (b), (c), and (d) we get the total deflection approximately.

(e) If damping was to be included we would use the velocity in (d), and hence find the damping forces and the deflections due to damping.

We proceed now to find a first approximation to the several deflections just referred to.

(*a*) The deflection due to the weight of the uniform beam is readily seen to be given by

$$\frac{24\,EI}{mg\,l^4} \cdot y = z(1 - 2z^2 + z^3).$$

Hence we have

TABLE XIII.

x	=	1	2	3	4	5	6	7
y	$\propto$	497	912	1185	1280	1185	912	497

The tabular figures are to be multiplied by $\dfrac{mgl^4}{3 \times 8^5 \times EI}$.

If for example $ml = 4M$ the tabular figures must be multiplied by

$$\frac{Mgl^3}{6 \times 8^4 \times EI}.$$

(*b*) The deflections due to the weight of the moving mass are as given already in Table IV if $P = Mg$.

The tabular figures must then be multiplied by $\dfrac{Mgl^3}{3 \times 8^4 \times EI}$ from paragraph 12. We may add the figures in Tables IV and XIII if we first multiply the former each by 2.

(*c*) Hence the deflections due to (*a*) and (*b*) are as in Table XIV when each is multiplied by $\dfrac{Mgl^3}{6 \times 8^4 \times EI}$.

TABLE XIV.

x_1 =	0	1	2	3	4	5	6	7	8
$x = 0$	0	0	0	0	0	0	0	0	0
1	0	595	659	687	685	659	615	559	0
2	0	1074	1200	1262	1264	1218	1136	1030	0
3	0	1375	1535	1635	1653	1599	1491	1347	0
4	0	1468	1632	1748	1792	1748	1632	1468	0
5	0	1347	1491	1599	1653	1635	1535	1375	0
6	0	1030	1136	1218	1264	1262	1200	1074	0
7	0	559	615	659	685	687	659	595	0
8	0	0	0	0	0	0	0	0	0

From this table we select the deflections corresponding to the several positions of M as it traverses the beam and enter them in Table XV.

The corresponding vertical velocities and vertical accelerations are obtained as tabular differences in the like manner as Tables V and VI were obtained from Table IV. These are also shown in Table XV.

TABLE XV.

For M.

x_1 =	0	1	2	3	4	5	6	7	8
Deflection . .	0	595	1200	1635	1792	1635	1200	595	0
Velocity . .	595	605	435	157	−157	−435	−605	−595	
Acceleration .	595	10	−170	−278	−314	−278	−170	10	595

Accordingly the accelerative forces on M when it is successively in the various positions x_1 are proportional to the figures in Table XVI.

TABLE XVI.

x_1 =	1	2	3	4	5	6	7
Force =	− 10	170	278	314	278	170	− 10

The corresponding deflections may be obtained by the tabular method, or more directly by multiplying the respective columns of Table IV by the corresponding accelerative forces in Table XVI.

We thus obtain

TABLE XVII.

Deflections due to acceleration of M.

x_1 = 0	0	1	2	3	4	5	6	7	8
$x = 0$	0	0	0	0	0	0	0	0	0
1	0	− 490	13770	26410	29516	22518	10030	− 310	0
2	0	− 810	24480	48650	55264	42534	19040	− 590	0
3	0	− 950	29750	62550	73476	57546	26010	− 810	0
4	0	− 940	29920	65052	80384	65052	29920	− 940	0
5	0	− 810	26010	57546	73476	62550	29750	− 950	0
6	0	− 590	19040	42534	55264	48650	24480	− 810	0
7	0	− 310	10030	22518	29516	26410	13770	− 490	0
8	0	0	0	0	0	0	0	0	0

(*d*) Now the bending moments and deflections due to the accelerative forces on the mass of the beam consequent on the moving force Mg are the same as those already obtained in Tables IV and XII, if $P = Mg$.

We may now proceed, if desired, to a second approximation by including the approximate values of the accelerative forces in the load system.

14. The scales of the tables are such that if T is any tabular figure then the value of the item is obtained by multiplying T by a suitable factor as given in the following list:—

Item.	Table No.	Value.
Mass per unit length of beam	—	m
Moving load	—	M
Deflection due to mg	XIII	$\frac{Tmgl^4}{3 \times 8^5 \times EI}$
Deflection due to Mg	IV	$\frac{TMgl^3}{3 \times 8^4 \times EI}$
If $ml = 4M$, deflection due to mg	XIII	$\frac{TMgl^3}{6 \times 8^4 \times EI}$
Deflection due to Mg and mg if $ml = 4M$	XIV	$\frac{TMgl^3}{6 \times 8^4 \times EI}$
Vertical velocity of M	XV	$\frac{TMgl^2v}{6 \times 8^3 \times EI}$
Vertical acceleration of M	XV	$\frac{TMglv^2}{6 \times 8^2 \times EI}$
Force due to acceleration of M	XVI	$\frac{TM^2glv^2}{6 \times 8^2 \times EI}$
B.M. due to acceleration of m	IV	$\frac{TmMgl^3v^2}{3 \times 8^4 \times EI}$
Deflection due to acceleration of M	XVII	$\frac{TM^2gl^4v^2}{18 \times 8^6 \times (EI)^2}$
Deflection due to acceleration of m	XII	$\frac{TmMgl^5v^2}{12 \times 8^6 \times (EI)^2}$

We are now in a position to add the deflections in Tables XIV, XVII, and XII. As a numerical example let it be given that $v = 60$ ft. per sec., $l = 100$ ft., $ml = 4M$, $EI = 35Mgl^2$.

Then, for example, when M is at the centre of the beam ($x_1 = 4$), the deflection at the centre ($x = 4$) is

due to mg $\quad 1280 \dfrac{mgl^4}{3 \times 8^5 \times EI} = \dfrac{25}{14}$ inch;

due to Mg $\quad 256 \dfrac{Mgl^3}{3 \times 8^4 \times EI} = \dfrac{5}{7}$ inch;

due to acceleration of M $80384 \dfrac{M^2 g l^4 v^2}{18 \times 8^6 \times (EI)^2} = \cdot 00157$ inch;

due to acceleration of m $6384 \dfrac{mMgl^5 v^2}{12 \times 8^6 \times (EI)^2} = \cdot 0007$ inch.

We observe in this example that the accelerative deflections are relatively small so that a further stage of approximation is unnecessary.

15. The methods that have been described and illustrated are simple and straightforward and not unduly long. Different terminal conditions can obviously be dealt with, and the process of finding approximately the equivalent of a particular integral of the equation of motion can be used with complicated systems of loading. It can also be applied to the solution of problems involving moving and pulsating forces or hammer blows. The methods are of general applicability.

No. 11.

INVESTIGATIONS OF THE CRYOSCOPY OF MILK.

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(Communicated by Professor J. Reilly.)

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Introduction.

One of the most important and difficult dairy problems confronting the public analyst is that of the detection of adulteration of milk with water. Thus, there is abundant evidence that perfectly genuine samples of milk may frequently fall below the legal limit of 8·5 per cent. in solids-not-fat, e.g. fully one-fourth of the individual samples reported on by Tocher (1) showing this behaviour; and, on the other hand, milk samples are frequently met with which are well above the minimum standard, and may suffer considerable dilution with water before arousing suspicion. Many suggestions for improved criteria of genuineness in milk have been proposed from time to time, such as the ash content of the milk, or the specific gravity or refractive index of the milk serum, but few offer any real advantage in cases where the composition of the original milk is unknown. Thus Tocher (1) finds as great a variation for ash content as for fat and solids-not-fat; Stubbs and Eldson (2) consider that the refractive index of the copper sulphate serum of the milk is in no way an improvement on the total-solids determination.

There is one determination, however, that of the freezing-point, which gives results of real value with practically all milks, and, since it has a definite physiological basis, can be taken as an almost absolute standard for the determination of adulteration, even when original samples are not available for comparison.

The skill and experience required in handling effectively the necessary apparatus have told somewhat against its use, however, and suggestions have been made from time to time, with the object of permitting the calculation of the freezing-point (or, alternately, the osmotic pressure, on which it depends) from the analytical data of milk, and thus achieving the same result indirectly.

The work of Mathieu and Ferrés on the simplified molecular constant (quoted in the Journal of Society of Chemical Industry, 1914 (3)) is an important attempt in this direction. The simplified molecular constant is the sum of the hydrated lactose and sodium chloride expressed as the isotonic equivalent of the lactose. It equals hyd. lactose + 11·9 × sodium chloride, since 1 g. of sodium chloride is the isotonic equivalent of 11·9 g. of lactose. It represents the joint effect of the two most important contributors, lactose and chlorides, to the freezing-point of milk. This constant has given rise to a voluminous literature in which it has been shown to vary more, however, than was originally supposed.

During a discussion on the freezing-point method (see Parker and Stackmann (4)) Monnier-Williams, referring to the work of Mathieu and Ferrés, suggested that it might be possible, by taking into consideration not only lactose and chlorides but other soluble constituents such as phosphates and citrates, to arrive at an expression which might show a much higher degree of constancy, comparable to that of the freezing-point. The present investigation is directed to an examination of this possibility.

Theoretical Basis for a Molecular Constant.

Porcher and Chevallier (5) have calculated the percentage of the freezing-point depression due to the various milk constituents in average milk to be as follows:—lactose, 53 per cent.; chlorides, 20 per cent.; soluble phosphates, 12 per cent.; citrates, 5 per cent.; non-protein nitrogen, 4 per cent.; and colloids, 5 per cent.

It is hard to say what signficance is to be attached to the colloid contribution of 5 per cent., which appears to be merely a suggestion by Porcher to cover the discrepancy between the freezing-point calculated from the dissolved constituents of milk, and that found in practice. It must be remarked, furthermore, that subsequent investigation has shown the osmotically active phosphorous compounds of milk to be more numerous than was thought (see Lenstrup (6), Lange and Miethke (7)). Nevertheless, Porcher's values may still be taken as affording a fairly useful basis for the consideration of the analytical values required for an approximate calculation of the freezing-point. Omitting the "colloid factor," it can be seen that lactose and chlorides together account for about 77 per cent. of the residual depression; lactose, chlorides, and phosphates about 89 per cent., and these with the addition of citrates nearly 95 per cent. It is evident that a molecular constant based on either of the latter two groups of constituents would be a definite improvement on the constant of Mathieu and Ferrés as a milk standard, and might approximate in value and in ease of obtaining to the freezing-point determination, if convenient and rapid methods for the determination of the necessary analytical data were available.

The constant of Mathieu and Ferrés in its present form requires the estimation of both chloride and lactose. The former is a simple and rapid estimation, the latter somewhat more troublesome, involving usually the use of the polarimeter. It is recognised that the readings of a much simpler instrument, the (dipping) refractometer, are in the main a measure of the lactose content of milk, and it was decided therefore to examine how far the use of this instrument, rather than the polarimeter, in conjunction with such further estimations as chloride and phosphate might meet the present case.

Lataix (8) has made a careful and detailed study of the refractometric method for milk, including a critical examination of the various methods in use for preparing the serum, which is valuable in this connection. His results on the actual and relative contribution of the various milk constituents to the refractive index of milk serum, and those of Porcher and Chevallier (5) on

their relative contribution to the freezing-point depression, are collected in the following table, which contains in addition, in the last two columns, some calculations based on these data :—

TABLE I.

Milk Constituent.	G. per l. (Porcher).	Act. F. P. dep. of Constituents.	Rel. cont. to F. P. dep. per cent.	Cont. to R. I. (Lataix).	Rel. cont. to R. I. per cent.	Rel. cont. to R. I. when Lactose cont. = 55·1	Extent to which R. I. falls short of F. P., *i.e.* Col. 3– Col. 6.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Lactose (hydrated) .	50	·293	55·1	·00730	83·6	55·1	—
Na, and K. Chlorides (as NaCl)	1·81	·120	22·6	·00038	4·3	2·8	19·8
Alkali Phosphates . . (= 0·96 P_2O_5)	2·1	·067	12·6	·00025	2·9	2·0	10·6
Citrates (of K. & Ca) .	3·25	·027	5·0	·00050	5·8	3·8	1·2
K_2SO_4 . . .	0·15	·004	0·8	·00010	1·1	0·7	0·1
$NaHCO_3$ . . .	0·25	·004	0·8	·00010	1·1	0·7	0·1
Non-Prot. N. (as urea)	0·50	·017	3·2	·00010	1·2	0·8	2·4
		– ·532	100·1	·00873	100·0	65·9	34·2

It is clear from column 5 that the refractive index is little more than an approximate method of estimating lactose. The last column, however, shows that it only requires the addition of the further estimation of chloride and soluble phosphate to that of the refractive index to approximate very closely to the freezing-point depression, as the influence of constituents (e.g. citrates) other than chlorides and phosphates is allowed for fairly well in the refractive index determination.

Suggested Molecular Constant.

It is possible to arrive at a theoretical connection between these three variables, refractive index, chloride, and soluble phosphate, aided by the data of Lataix and Porcher. The augmentations of the refractive index over that of water for solutions (1 g. per 100 g. of water) of lactose (hyd.), sodium chloride, and soluble alkali phosphates of pH 6·6 (as P_2O_5), expressed as Zeiss numbers, i.e. as values on the scale of the Zeiss dipping refractometer, are 3·48, 4·52, and 7·14 respectively (cp. Rothenfusser and Kotschopoulos (11)), and the corresponding freezing-point depressions, 0·052, 0·615, and 0·585° C. respectively.

If now we take a milk containing L g. lactose, C g. chloride (as NaCl), and P g. of soluble phosphate (as P_2O_5) per 100 g. water, the following approximate equations hold :—

(*a*) $$R_s - R_w = 3{\cdot}48L + 4{\cdot}52C + 7{\cdot}14P + K_1,$$

where R_s and R_w represent the refractive indices of milk serum and of water in degrees Zeiss.

(*b*) Freezing-point, $\triangle = 0{\cdot}052L + 0{\cdot}615C + 0{\cdot}585P + K_2$,
where K_1 and K_2 are relatively small quantities representing the contribution of other less important milk constituents. Eliminating L between the equations gives

$$66{\cdot}9 \triangle = (R_s - R_w) + 36{\cdot}6C + 32{\cdot}0P + K_3 \ldots (1).$$

The value of K_3 will be relatively small, much more so than that of either K_1 or K_2, for these factors tend to cancel out in the elimination of L from the two equations, as is evident from a comparison of the data of the last column of Table I with those of Columns 3 and 6.

The expression $(R_s - R_w) + 36{\cdot}6C + 32{\cdot}0P$ for a sample of milk may therefore be expected to be very approximately proportional to its freezing-point, and to afford thus an improved molecular constant applicable to all *fresh* samples of milk.

Analytical Methods—Refractive Index.

The convenience and rapidity in use of the dipping refractometer established it as the most suitable type of instrument for general use, and the only question which arose was, therefore, that of choice of serum. The work of Lataix (8) had narrowed down the choice to two or perhaps three, namely, Ackermann's, in which protein is removed by boiling with concentrated $CaCl_2$; that of Ambuhl and Weiss, in which acid mercuric chloride is employed in the cold; and Lythgoe's well-known copper sulphate serum. It was felt that the two chief determinations, namely, those of refractive index and chloride, should, if possible, be made on the same serum, to avoid having to correct either value for protein and fat content of the milk sample, and that its preparation should not involve the use of strongly acid reagents, on account of the danger of bringing additional phosphate, etc., into solution.

The Ackermann serum fulfilled these conditions, and its only disadvantage, that of containing a considerable amount of added chloride (from $CaCl_2$), which would interfere with its employment for the simultaneous determination of both refractive index and chloride, was easily overcome by using the equivalent amount of calcium nitrate for the protein precipitation instead of the chloride.

In the modification adopted the serum is prepared by adding to 30 c.c. milk (contained in a small flask) 0·25 c.c. of an approximately 47·5 per cent. solution of $Ca(NO_3)_2$, $4H_2O$. The milk is thoroughly mixed with the nitrate, and the flask then closed by a cork, through which was passed a short piece of glass tubing to act as a condenser. The flask is immersed in boiling water for 15 minutes with occasional gentle agitation, and then cooled. After mixing the contents carefully with the condensed water on the side of the flask and tube the serum is obtained by decantation or filtration. The calcium nitrate solution used, diluted ten-fold, should have a Zeiss No. of 26 at 17·5° C., at

which the reading for water should be 15·0. This was also checked on each occasion.

Determination of Chloride.

Numerous methods, gravimetric and volumetric, are available for this determination. As the result of a systematic examination Kieferle (9) recommends for general use the very simple and direct Drost method. In this the Volhard process is used directly on the milk, the interference of protein, and the commonly disturbing back reaction between silver chloride and ferric thiocyanate, which causes a fading of the end point, being overcome by the presence of a considerable excess of nitric acid. The method was found in the course of the present work to be very satisfactory and easy to carry out, especially on the Ackermann serum.

To 10 c.c. of serum are added 5 c.c. of approx. 5 N/1 HNO_3, then 5 c.c. of N/10 silver nitrate (add slowly and mix gently with rotary motion whilst adding), and finally about 1–2 c.c. of a fairly strong solution of ferric sulphate. Titrate excess of N/10 silver nitrate with N/10 potassium thiocyanate.

Estimation of Soluble Phosphate.

The devising of a rapid and convenient method for the estimation of the soluble phosphate in milk presented some difficulty. Various sera, e.g. Ackermann's, Schwartz's with methyl alcohol, etc., were tested for the purpose, but in no case found satisfactory. Rennet serum was indeed suitable, as it is largely free from insoluble or colloidal phosphates, which are removed in the casein clot, but was somewhat too tedious to prepare to be convenient. Finally, however, a fairly rapid method for the approximate determination of soluble phosphate was developed which was directly applicable to the milk sample. It is hoped to deal more fully with this and related matters on another occasion, and the procedure adopted will only be outlined here.

Briefly, it consists in the addition to milk of a small amount of a soluble oxalate (slightly less than is sufficient to remove the soluble calcium), neutralising to phenolphthalein, and then adding a known excess of standard alkali, followed by a little strong calcium chloride solution. The excess alkali is then back titrated to the original phenolphthalein tint with standard acid. At the first neutralisation the soluble phosphate is present mainly as K_2HPO_4, and this is converted by the successive addition of extra alkali and calcium chloride into K_3PO_4 and insoluble $Ca_3P_2O_8$ respectively. The difference between the back titration and the excess alkali added thus measures the alkali that was used in converting secondary to tertiary phosphate according to the equation—

$$K_2HPO_4 + KOH = K_3PO_4 + H_2O,$$

and can, therefore, be used to estimate the soluble phosphate.

To 10 c.c. of milk add 1 drop of saturated neutral potassium oxalate and 5 drops of a 0·5 per cent. alcoholic solution of phenolphthalein, neutralize with N/10 caustic soda to standard pink colour, approximately pH 9·3 (standard

colour produced by adding 1 drop of dilute (·01 per cent.) magenta to 10 c.c. of milk); then add a further 2 c.c. N/10 caustic soda, 1 c.c. of M/1 calcium chloride, allow to stand for one minute, and titrate back to the original tint with N/10 H_2SO_4. The difference between the alkali added (2 c.c.) and the latter titration (corrected for any blank due to CO_2 in the N/10 caustic soda) gives a measure of the soluble phosphate, 1 c.c. N/10 alkali being equivalent to ·0071 g. P_2O_5.

The method has been checked with a known solution of phosphate, and its applicability to milk verified by numerous comparisons with the results of similar estimation on the corresponding wheys.

Convenient methods for all three estimations, viz.: refractive index, chloride, and soluble phosphates, were now available, and the next stage—the practical testing of Equation 1—could be proceeded with.

Check of proposed "molecular constant."

It was seen earlier from theoretical considerations, based mainly on the work of Porcher and Lataix, that the expression

$$(R_s - R_w) + 36{\cdot}6C + 32{\cdot}0P = K\triangle,$$

where K is a constant, and the other symbols have the meanings already given.

As it was intended to estimate chloride by titration with N/10 $AgNO_3$ in 10 c.c. of the modified Ackermann serum, and phosphate by the use of N/10 NaOH per 10 c.c. of milk, it was convenient to rewrite the above equation directly in terms of these titration values. Now 1 c.c. N/10 $AgNO_3$ = ·00585 g. NaCl ∴ 36·6C becomes 2·2 × (c.c. N/10 $AgNO_3$), and 1 c.c. N/10 NaOH = ·0071 g. P_2O_5 ∴ 32·0P becomes 2·3 × (c.c. N/10 NaOH) ∴ $(R_s - R_w) + 2{\cdot}2 \times$ (c.c. N/10 $AgNO_3$) + 2·3 × (c.c. N/10 NaOH) = K△ . . . . (2). A slight correction is still necessary in this equation. The phosphate titration is made on the original milk, and the other two estimations on the Ackermann serum, so that they are not directly comparable without correcting for the volume of fat and protein precipitated by the Ackermann reagent. The difference is not very important, and it is sufficient at most to correct for the general case of an average milk. Assuming for this 3·75 per cent. fat of density 0·93, and 3·0 per cent. precipitable protein of density 1·65, the phosphate factor 2·3 becomes —

$$2{\cdot}3 \times \frac{30}{30{\cdot}25} \times \frac{100}{100 - 3{\cdot}75/{\cdot}93 - 3{\cdot}0/1{\cdot}65}, \quad \text{or } 2{\cdot}4 \text{ approx.}$$

(Here 30/30·25 represents the degree of dilution in the preparation of the Ackermann serum due to addition of calcium nitrate solution.) The final form of the equation is therefore—

$(R_s - R_w) + 2{\cdot}2 \times$ (c.c. N/10 $AgNO_3$) + 2·4 (c.c. N/10 NaOH) = K△ . . . (3),

where the chloride is determined on 10 c.c. of serum and the phosphate on 10 c.c. of milk.

The refractive index (Zeiss No.), chloride, and phosphate were determined for a number of individual and mixed milks, using the methods set out, and

making all estimations in duplicate. The freezing-point was determined by the Hortvet instrument, allowing the same amount of supercooling (1·25° C.) in all cases, and determining the freezing-point of water on each occasion. Table 2 shows results of these estimations for individual milks (Nos. 1 to 26) and mixed milks (A to K), together with the "molecular constant" (K △) calculated from the equation given above. The final column shows the ratio between the "molecular constant" and the freezing-point for each sample.

TABLE 2.

Sample No.	R. I. Serum. (R_s)	R. I. Water. (R_w)	$R_s - R_w$.	Chl. in c.c. N/10 $AgNO_3$ × 2·2.	Phos. in c.c. N/10 NaOH × 2·4.	K △ calc. sum of cols. 4 + 5 + 6.	F. P.	K△/F.P. = K.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	39·60	14·90	24·70	5·17	3·60	33·47	−0·561	59·66
2	39·55	15·15	24·50	7·19	2·64	34·33	−0·563	60·58
3	39·65	15·05	24·60	5·28	3·00	32·88	−0·544	60·44
4	40·30	15·10	25·20	5·21	3·18	33·53	−0·548	61·18
5	39·80	15·10	24·70	4·95	3·19	32·84	−0·537	61·15
6	39·10	15·00	24·10	6·67	2·11	32·88	−0·543	60·56
7	38·90	15·15	23·75	5·83	3·53	33·11	−0·542	61·08
8	40·90	14·95	25·95	4·97	2·83	33·55	−0·553	61·02
9	39·65	15·00	24·65	5·94	3·12	33·71	−0·545	61·85
10	39·90	14·95	24·95	6·71	1·92	33·58	−0·554	60·61
11	40·95	14·90	26·05	4·84	3·40	34·29	−0·568	60·38
12	40·00	14·90	25·10	4·91	3·57	33·58	−0·547	61·40
13	40·35	15·15	25·20	5·32	3·04	33·56	−0·552	60·78
14	39·45	15·10	24·35	6·09	3·40	33·84	−0·551	61·41
15	40·00	15·00	25·00	5·28	2·97	33·25	−0·547	60·79
16	39·60	15·00	24·60	5·61	3·24	33·45	−0·549	60·93
17	38·75	15·00	23·75	6·27	2·93	32·95	−0·545	60·20
18	39·25	15·00	24·25	4·84	3·38	32·47	−0·545	59·63
19	40·00	15·00	25·00	3·96	3·79	32·75	−0·546	60·21
20	39·95	15·00	24·95	4·40	3·33	32·68	−0·544	60·07
21	39·95	15·00	24·95	4·84	3·28	33·07	−0·542	61·01
22	38·45	15·00	23·45	6·16	3·19	32·77	−0·543	60·40
23	36·95	15·00	21·95	8·10	2·64	32·69	−0·540	60·54
24	33·95	15·00	18·95	10·89	2·52	32·36	−0·537	60·26
25	36·05	15·00	21·05	9·31	1·96	32·32	−0·533	60·64
26	35·00	15·00	20·00	10·56	2·28	32·84	—	—
A	40·30	15·00	25·30	5·61	2·93	33·84	−·553	61·19
B	39·60	15·00	24·60	5·53	2·91	33·01	−·545	60·57
C	38·80	15·00	23·80	6·27	2·59	32·66	−·545	59·97
D	39·45	14·90	24·55	5·50	2·67	32·72	−·544	60·14
E	38·90	15·00	23·90	6·38	2·52	32·80	−·545	60·18
F	39·70	15·00	24·70	5·65	2·93	33·28	−·548	60·72
G	39·65	15·00	24·65	6·16	1·83	32·64	−·538	60·67
H	39·00	15·00	24·00	6·31	2·52	32·81	−·541	60·68
I	39·45	15·00	24·45	6·16	2·64	33·25	−·542	60·22
J	39·35	15·00	24·35	6·09	2·64	33·08	−·548	60·37
K	39·20	15·00	24·20	6·16	2·69	33·05	−·543	60·87
				Mean values,		33·14		60·60

From this table it will be seen that the value of the "molecular constant" lies within a narrow range, viz., from 32·32 to 34·33, having an average value of 33·14. In no case does the molecular constant differ by more than 3·5 per cent. from the average value,[1] and this degree of variation is no greater than that shown by the freezing-points themselves (3·7 per cent.). Some of the milks which had very low "Zeiss Numbers," and on this criterion alone might be suspected to have been diluted, gave at the same time values close to the average for the "molecular constant," e.g. samples Nos. 23 to 26 (inclusive), which in addition to low refractive index gave solids-not-fat content well below the legal standard.

The "molecular constant" compared with the corresponding freezing-point gives a ratio lying between the limits of 59·63 and 61·85, having an average value of 60·60. The ratio in no case differs by more than 2 per cent. from the average.

The results for the *mixed* milks alone (A to K) show, as might be expected, the range of the "molecular constant" to be even narrower, viz., from 32·64 to 33·84, with an average value of 33·01. In no case does the constant differ from the mean value by more than 2·5 per cent. For the ratio of the "molecular constant" to the freezing-point depression the values are even closer, ranging from 59·97 to 61·19, and thus lying within 1 per cent. of the average 60·52.

From these results it seems that the genuineness of a sample of milk can be determined by the use of this "molecular constant" with almost the same degree of certainty as by the freezing-point method. It would, of course, be necessary to examine a very large number of samples before fixing average and minimum standards.

The present results from nearly forty different samples (mainly individual milks) allow provisional standards to be drawn up, however, which it is hoped to test more fully later on. As a preliminary the lowest value found, viz., *32·32* (No. 25, Table 2), may be taken as a minimum (this belongs to an individual milk of very low freezing-point depression, and is likely to be exceeded by any genuine value found for mixed milk), and the average value of the molecular constant for all samples accepted as *33·1*. From these figures the amount of water which could be added to an average milk without bringing its "constant" below the suggested minimum is about 3 per cent., i.e. an allowance similar to that usually made when the freezing-point method is employed.

A few determinations of added water have been made based on the "molecular constant" for various mixed milks to which known amounts of water were added. The formula used is $W = 100 \times \frac{33{\cdot}1 - (K\triangle)}{K\triangle}$, where W is the

[1] If the quantity $(R_s - R_w) + 2{\cdot}2$ (c.c. N/10 $AgNO_3$), omitting the phosphate term, is taken instead of the full expression, a simpler, if less accurate, type of molecular constant, somewhat similar to that of Mathieu and Ferrés, is obtained which might be useful in certain cases. For the samples in Table 2, this expression gives a range of from 28·96 to 31·69, mean value 30·23, and thus an extreme variation from the mean of about 5 per cent.

percentage of added water, K△ the molecular constant found, and 33·1 its average value for genuine milk. For five milk samples to which 5, 10, 10, 15, and 20 per cent. of water had been added, the amounts found were 6·4, 11·1, 11·2, 16·3, and 20·6 respectively, a very satisfactory agreement. Of course, in dealing with completely unknown samples, a deduction of 3 per cent. from the added water found would be made, as pointed out above.

It seems justifiable to conclude that the application of the "molecular constant" derived from the combined values of the refractive index, chloride, and phosphate, is a satisfactory criterion for detecting added water in a sample of milk, and estimating its approximate amount.

The method is fairly rapid and easy to carry out, and might be particularly useful where the refractive index is used as a routine method. (See Hanley, World's Dairy Congress Report, 1928 (10).) For doubtful samples, the extra estimations of chloride and phosphate would give little trouble, being very much easier in such cases than an additional freezing-point determination. It must, of course, be emphasized that the results in their present form apply only to fresh milks. It is hoped to deal with the possibility of extension to sour samples in a later communication.

Summary.

(1) A cryoscopic constant is developed which takes account of the chief osmotic substances of milk, viz., lactose, chloride, and phosphate (also citrates, greater part).

(2) Methods for the various analytical determinations required therefor, viz., refractive index, chlorides, and soluble phosphate, are discussed. The calcium nitrate serum is selected as the most suitable for refractive index and chloride estimations, and a rapid and convenient method suggested for the approximate determination of soluble phosphate by direct titration of milk.

(3) Preliminary work on the relation of the proposed constant to the freezing-point of fresh milk, mixed, individual samples, and samples of low specific gravity, shows the correspondence between the two to be very close. For some 40 samples examined the constant itself lay within a narrow range, viz., from 32·32 to 34·33 for individual samples (including those of low specific gravity), and from 32·64 to 33·84 for samples of mixed milks, the average value in the case of individual samples being 33·19 and in mixed samples 33·01. The lowest value occurred in an individual sample, viz., 32·32, which diverged less than 3 per cent. from the average.

(4) The determination of the "constant" in the case of milks to which known amounts of water had been added, permitted very satisfactory calculation of the degree of dilution. It is suggested that this estimation may be more convenient than that of the freezing-point in many cases, especially where determination of refractive index is already used as a routine method.

References.

1. Tocher (1925).—"Variations in the Composition of Milk." H.M. Stationery Office, Edinburgh.
2. Stubbs and Eldson (1930).—Analyst, vol. **55**, 618.
3. Mathieu and Ferrés (1914).—Abstr. in Jour. Soc. Chem. Ind., vol. **33**, 214.
4. Parker and Stackmann (1929).—Analyst, vol. **54**, 217.
5. Porcher and Chevallier (1923).—Le Lait, vol. **3**, 188.
6. Lenstrup (1926).—Jour. Biol. Chem., vol. **70**, 193.
7. Lange and Miethke (1932).—Milch. Forsch, vol. **14**, 195.
8. Lataix (1929).—Le Lait, vol. **9**, 474, 612, 711, 844.
9. Kieferle (1928).—Milch. Forsch, vol. **5**, 532.
10. Hanley (1928).—Rept. World's Dairy Congress, London, 744–767.
11. Rothenfusser and Kotschopoulos (1933).—Milch. Forsch, vol. **16**, 80.

No. 12.

THE INFLUENCE OF THE STAGE OF LACTATION ON FAT ESTIMATIONS BY THE GERBER METHOD.

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(Communicated by Mr. E. J. Sheehy.)

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That the average size of the fat globules found in milk decreases, and that the smaller size globules increase considerably in number as the lactation period advances, has been shown by Campbell (1), Rahn and Sharp (2), Weigmann (3), and others.

That the smaller fat globules separate more slowly from the serum in which they are suspended than the larger ones is evident from mathematical considerations, and the velocity of travel of different size globules at varying distances from the centre of revolution has been calculated by Rahn and Sharp (4).

It has also been shown by the latter workers that, while the number of fat globules 2μ and less in diameter found in separated milk constitutes about 89 per cent. of the total, yet these make up only about 42 per cent. of the total weight. The fat content of the separated milk investigated was, however, very high (·13 per cent.), and in separated milk with normal fat content (say, ·05 per cent.) a greater proportion of the total weight would, undoubtedly, be made up of globules 2μ and less in diameter. It is manifest, however, that with any centrifugal method the small globules, particularly those 2μ or less in diameter, are not easily separated.

It is, therefore, logical to assume that when the fat content of milk is estimated by the Gerber method, run at the usual speed and for the usual time (3 to 5 minutes), the accuracy of the estimations will vary with the relative proportions of the small fat globules present in the samples, and that accurate estimations of the fat content of milk from cows advanced in their lactation can only be obtained by whirling the samples for a longer time or at a higher speed, owing to the relatively high number of small fat globules present.

The standard method of operating the Gerber test for estimating the fat content of whole milk is briefly as follows (5):—Shake test bottles containing milk, sulphuric acid, and amyl alcohol, until the curdy matter is fully dissolved. Place bottles in centrifuge immediately, and while still hot whirl for 3 minutes

at a speed of 800 to 1,200 revs. per minute. Stop centrifuge, and place test bottles in water bath at 149° F. Read off at this temperature.

It has been shown by Lyons and Finlay (6) that in the case of separated milk in which the fat is present mainly in the form of small globules at least 3 whirlings of 5 minutes each, with heating of test bottles in between, are necessary to give readings which agree reasonably with those obtained by the Roese Gottleib method, and that in the case of separated milk investigated (i.e., from cows in the late stage of lactation) only about $\frac{1}{4}$ of the fat was separated by one whirling period of 5 minutes.

It has been shown by Campbell (1) and Weigmann (3) that the proportion of small fat globules in milk, as well as the mean average volume of all fat globules, varies with the feed, the breed, and the individuality of cows. This would obviously cause variation in the error obtained in estimating the fat content of their milk by the standard Gerber method even towards the end of their lactation periods. The results of the following experiments show that, although such variation occurs, the error increases in all cases as the lactation advances.

For our purpose, 25 grade shorthorn cows at varying stages of their lactation ranging from 4 to 49 weeks were selected. All were treated alike as regards feed. Samples of each cow's milk were taken and tested in quadruplicate by the Gerber method. The tests were read off after one whirling of 3 minutes, and again after 3 whirlings of 3 minutes, the test bottles having been heated to 160° F. between whirlings. The readings obtained were checked against Roese Gottleib estimations for the same samples. Certified Gerber test bottles, such as are generally used for whole milk testing, were employed in the first instance, but as it was found difficult to estimate the fat percentage closer than ·05 per cent. with these, the estimations were repeated, using "precision" bottles capable of being read accurately to ·01 per cent.

Fat globule counts were made of all samples so as to determine the correlation between fat globule size and the under-reading obtained by the Gerber method when used in the ordinary way, i.e., when the bottles were whirled for three minutes only.

The counts and sizes of the fat globules for the different stages of the lactation period were estimated according to the following method:—The samples of milk were diluted with a 3 per cent. gelatine solution, so as to eliminate Brownian movement. A portion of each was placed in a Zeiss blood-cell counting chamber, and the cover slip vaselined around the edges, to prevent flowing motion. The microscope used, fitted with a 1/12″ oil immersion lens and a 5 × micrometer eye-piece, gave such a magnification that each division of the eye-piece micrometer scale was equivalent to 1μ on the objective. This was verified by checking against a micrometer objective graduated to read 1/100 mm. or 10μ. The slides were made to travel in a straight line by moving the mechanical stage, and the eye-piece micrometer scale was set perpendicular to the line of movement, so as to facilitate measurement. Three slides were

made for each sample, and the size of all globules which crossed the scale in the course of travel were noted until 1,000 globules had been measured for each sample.

Influence of the Stage of Lactation on the Size of Fat Globules.

Table 1 gives the number of globules falling in the different groups according to size per 1,000 globules counted at 25 different stages of lactation, ranging from 4 to 49 weeks, and also the increase obtained in the fat reading (using precision test bottles) by whirling the bottles for 3 periods of 3 minutes each (heating to 160° F. in between) over whirling once only for 3 minutes.

TABLE 1.

Variation of Globule Size with Stages of Lactation.

Stages of Lactation	Globule sizes grouped					TOTAL.	Fat reading increase 3 whirlings over 1.
Weeks.	0–2μ	2–4μ	4–6μ	6–8μ	8–10μ		
4	184	416	336	58	6	1000	·035
4	162	390	380	62	6	1000	·022
5	160	414	350	68	8	1000	·025
6	171	383	381	58	7	1000	·025
8	184	419	341	54	2	1000	·047
11	260	406	284	44	6	1000	·051
12	235	422	296	43	4	1000	·042
12	277	399	269	52	3	1000	·050
15	249	412	285	49	5	1000	·047
18	283	400	270	45	2	1000	·057
20	265	514	191	26	4	1000	·078
22	283	527	169	18	3	1000	·063
25	265	568	156	11	–	1000	·072
27	297	560	137	6	–	1000	·050
28	301	531	163	5	–	1000	·067
34	381	504	97	18	–	1000	·097
35	406	526	60	7	1	1000	·102
37	401	481	106	11	1	1000	·132
38	333	515	148	4	–	1000	·100
39	311	447	207	30	5	1000	·097
40	513	439	44	4	–	1000	·160
41	414	423	129	27	6	1000	·152
44	352	511	128	9	–	1000	·115
45	454	473	65	7	1	1000	·132
49	415	481	101	11	–	1000	·140

It will be seen from the foregoing Table that globules with diameters less than 2μ tend to increase in number, and those 4μ and upwards in diameter tend to decrease in number as the lactation period advances. The group of globules $2–4\mu$ in diameter tend to vary little, and somewhat irregularly. The fat reading obtained by whirling for three 3-minute periods shows a continuous increase over that obtained by a single whirling for 3 minutes as the lactation period advances, and a distinct correlation is noticeable between this increase and the increase in the number of fat globules 2μ or less in diameter.

To eliminate the influence of the individuality of the cows on the size of the fat globules and to obtain average figures, the lactation stages were grouped, such as shown in Table 2.

TABLE 2.

Stage of Lactation. Weeks.	No. of Cows Avgd.	Globule sizes grouped according to diameter. $0–2\mu$	$2–4\mu$	$4–6\mu$	$6–8\mu$	$8–10\mu$	Total.	Fat reading increase 3 whirlings over one. %
0–9	5	172	404	358	60	6	1000	·031
10–19	5	261	408	281	47	4	1001	·051
20–29	5	282	540	163	13	1	999	·066
30–39	5	366	495	124	14	1	1000	·105
40–49	5	430	465	93	12	1	1001	·140

The number of globules grouped according to diameter does not give a clear conception of the quantity of fat present in the form of globules of the different sizes, and for this reason Table 3, showing the percentage by volume present under the different groups, is given. The volume is calculated according to formula :—

$$\frac{N \times \cdot 5236\ D^3 \times 100}{S}$$

which gives the percentage volume of fat under each globule group when N is the number of fat globules in group, and S the sum of the volumes of all groups for the particular lactation stage under consideration.

For the purpose of the calculation the mean diameter for each group was taken. It is realised that this is only accurate when the globules of different sizes within the group are distributed over the diameter range between the extremes, so as to give the same volume for every size. To what extent this may be so is difficult to judge, but proceeding upwards on the scale of diameter the diminishing numbers will tend to minimise the error. In the case of the smaller globules, i.e., 2μ or less in diameter, the number is apparently increasing

as the diameter scale within the group is ascended, and taking this average at 1μ would lead to an underestimation of the volume. It is realised also that many very small globules may escape detection by the method employed, but the error arising from this would not be considerable.

Table 3.

Stage of Lactation Weeks.	No. Avgd.	Percentage fat volume under different groups of fat glob.					Total.	Fraction of A in Avg. Fat tests.	Average Fat Test.
		0-2μ A	2-4μ	4-6μ	6-8μ	8-10μ			
0–9	5	·21	13·50	55·39	25·49	5·41	100	·0076	3·641
10–19	5	·39	16·83	53·68	24·64	4·46	100	·0146	3·742
20–29	5	·70	36·07	50·40	11·03	1·80	100	·0282	4·034
30–39	5	1·05	38·45	44·59	13·81	2·10	100	·0420	3·998
40–49	5	1·46	42·62	39·47	13·97	2·48	100	·0623	4·268

It appears from the following Tables that the increase obtained from three whirlings for three minutes each is greater for all stages of lactation than the above fractions for globules of 2μ, and less in diameter of the average fat tests, indicating that either one whirling fails to separate out some globules over 2μ in diameter or that the volume of globules under 2μ in diameter is considerably underestimated.

Influence of Stage of Lactation on the Fat Reading by the Gerber Method.

The Gerber test bottles used had been tested and found correct in the State Laboratory, Dublin. They were again re-checked in the College Laboratory. The milk samples were measured in all cases with the same pipette. Quantities of milk of specific gravity 1·032 delivered by this pipette were weighed, and were found to average 11·32 g. with a maximum variation of ·005 g. The correct fat percentages were determined, therefore, by correcting the fat readings observed to those which would have been obtained on 11·25 g. of milk.

To determine whether the ordinary method of using the Gerber test (i.e., whirling for 3 minutes) was sufficient to show up all the fat, the test bottles were whirled for one 3-minute period and read off, and then whirled for two further 3-minute periods, being heated to 160° F. between the whirlings, and again read off—the increase due to the additional whirling being noted. All tests were read off at 149° F. The whirling periods were 3 minutes at full speed of the centrifuge, which was 985 r.p.m. All tests were made in quadruplicate and averaged.

Table 4 shows the readings obtained by whirling tests for 1 3-minute

period and 3 3-minute periods, with the increases obtained for the latter. These tests were made with the ordinary Gerber test bottles (certified).

TABLE 4.

Stage of Lactation in Weeks.	No. of Cows Avgd.	Fat Percentage. Whirled 1 period.	Whirled 3 periods.	Average Increase.
0–9	5	3·641	3·641	None
10–19	5	3·742	3·742	"
20–29	5	4·034	4·034	"
30–39	5	3·933	3·998	·065
40–49	5	4·173	4·268	·095

The additional whirlings appeared to give slight increases for the milk of cows in the early stages of their lactation, but these were so small (less than ·05 per cent.) that they could not be read with any degree of accuracy on the ordinary milk bottle.

To obtain a more accurate estimation of the difference between readings after one and three whirlings, a special type of test bottle used for estimating fat in skim milk ("precision" test bottle) was employed, and all estimations repeated in quadruplicate, and in accordance with the procedure already set out. With these bottles it was possible to read to ·01 per cent. Table 5 shows the differences obtained by this method.

TABLE 5.

Stage of Lactation in weeks.	No. of Cows Averaged.	Average increase three whirlings over one.
0–9	5	·031
10–19	5	·051
20–29	5	·066
30–39	5	·105
40–50	5	·140

The differences shown in Tables 1 and 2 are those obtained by the use of this type of test bottle.

The results obtained by the Gerber method, using the ordinary certified milk test bottles, were compared with Roese Gottleib estimations for the same samples.

The Roese Gottleib estimations were carried out in accordance with the procedure detailed by Majonnier and Troy (7), and the weight of milk taken for each estimation was determined to ·005 g. All estimations were made in

duplicate. The comparison of the Roese Gottleib test and those of the Gerber test for one and three whirlings are shown in Table 6.

TABLE 6.

Lactation Stage.	Roese Gottleib Fat %.	Gerber 1 Whirling.	Gerber 3 Whirlings.	Excess of Roese-Gottlieb over Gerber at 1 Whirling.	Excess of Roese-Gottlieb over Gerber at 3 Whirlings.
0–9	3·646	3·641	3·641	·005	·005
10–19	3·753	3·742	3·742	·011	·011
20–29	4·042	4·034	4·034	·008	·008
30–39	4·012	3·933	3·998	·079	·014
40–49	4·269	4·173	4·268	·096	·001

When the percentage of fat by volume in the form of globules 2μ and less in diameter, shown in Table 3, is plotted against the stage of lactation and the curve so obtained compared with that obtained by plotting the increase in the fat readings by the Gerber method for three whirlings over one, the correlation between the number of small fat globules in the milk and the extent of the under-reading obtained when the tests are read after one 3-minute whirling period is manifest. (See fig. 1.)

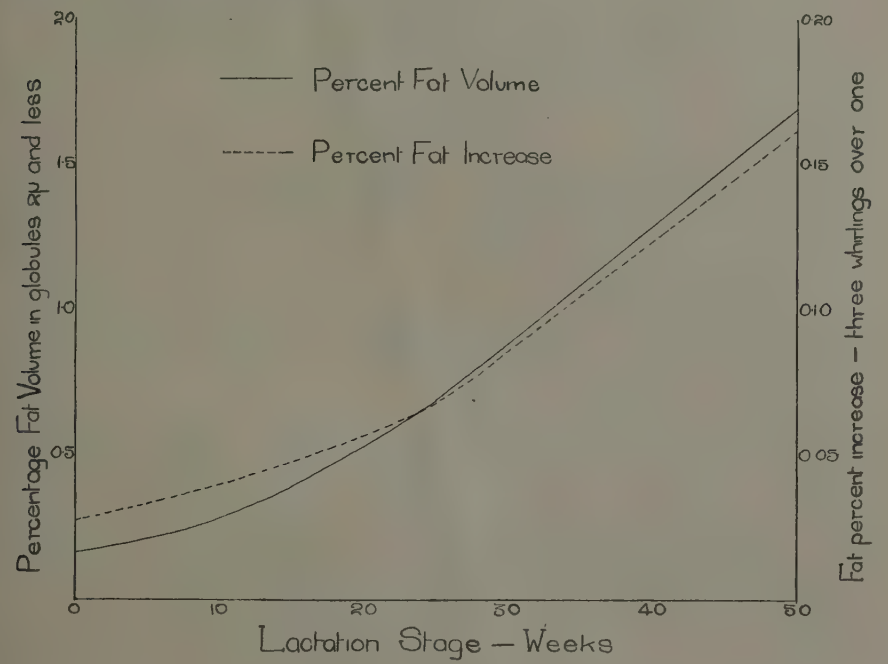

FIG. 1.

Conclusion.

The underestimation of the fat percentages which takes place towards the end of the lactation, when the tests are whirled for one 3-minute period only, would not have any great significance in creamery practice, nor would it to any extent affect the equity of payments for milk when these are calculated on a butterfat basis. The fraction of the fat not shown up by one whirling being mainly in the form of very small globules, would, in any case, be mostly lost in the manufacturing process, i.e., in separating and churning, and would not help the creamery to obtain an increased over-run.

The matter would have greater significance in connection with cow-testing, particularly in the case of cows barely able to reach the butterfat standard for registration, which is in this country 225 lbs. in 45 weeks (8).

Fig. 2 gives a lactation curve for ten cows which would just about reach the qualifying standard. The figures were taken from Cow-Testing Association

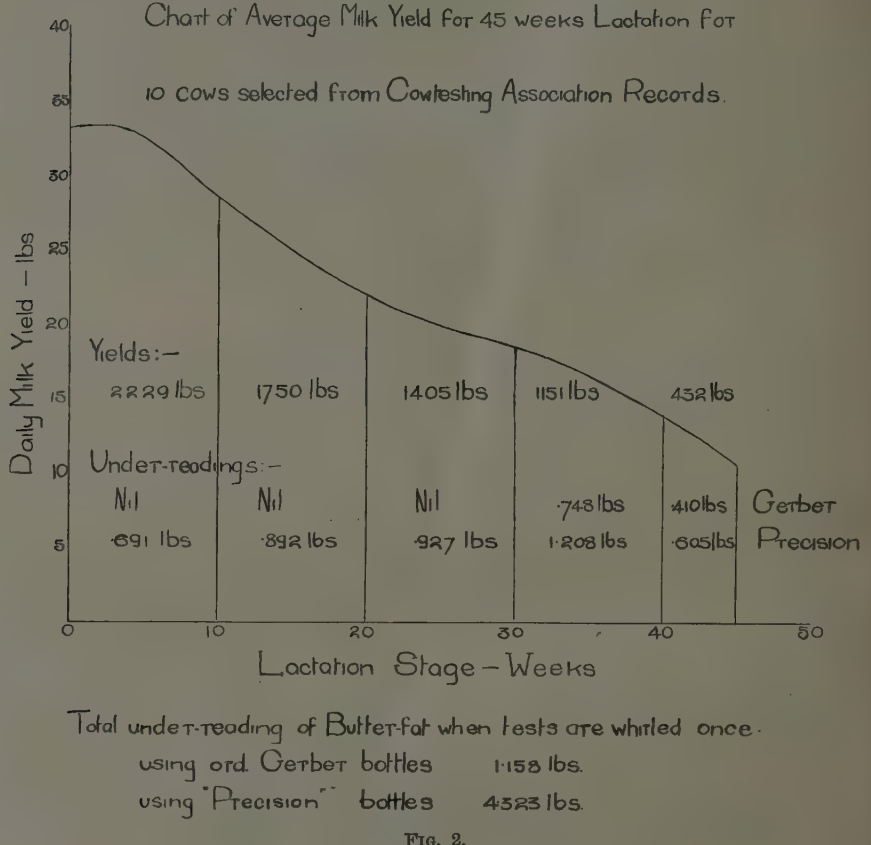

Fig. 2.

records and averaged. The underestimation of fat, when the tests were made with the ordinary Gerber test bottles only whirled for a 3-minute period, is about 1¼ lbs. of butterfat for a 45 week period; but, according to the more accurate results obtained by the "precision" test bottles, it is about 4¼ lbs.

In the case of cows yielding milk containing a high proportion of small fat globules the underestimation may greatly exceed these figures. Therefore, when the test bottles are whirled for 3 minutes only, particularly towards the end of the lactation period, cows may be unjustly disqualified for registration, and this applies more particularly to cows that yield milk containing a high proportion of small fat globules.

REFERENCES.

1. CAMPBELL, M. H.—A Study of Fat Globule Size in Milk. Vermont Ag. Expt. Stn., Bull. 341, 1932.
2. RAHN, O., and SHARP, P. F.—Physik der Milchwirtschaft, p. 41, 1928.
3. WEIGMANN, H. H.—Ueber den Despersionsgrad des Fettes in der Milch. Milchwirtschaf Forsch., pp. 259–311, 1926.
4. RAHN, O., and SHARP, P. F.—Physik der Milchwirtschaft, p. 69, 1928.
5. DEPARTMENT OF AGRICULTURE, IRISH FREE STATE.—The Estimation of Fat in Milk and Cream, Bull. 4, 1932.
6. LYONS, J., and FINLAY, W.—Influence of Heating and Agitating Milk before Separation on the Fat Loss in the Skim Milk. Econ. Proc. Royal Dublin Society, Vol. 2, No. 27, 1932.
7. MAJONNIER and TROY.—Technical Control of Milk Products, p. 108, 1925.
8. DEPARTMENT OF AGRICULTURE, IRISH FREE STATE.—Scheme for the Registration of Non-Pedigree Cattle. Scheme No. 20, 1933.

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